Quarterly Journal of the Royal Meteorological Society Q. J. R. Meteorol. Soc. 141: 249–257, January 2015 A DOI:10.1002/qj.2354

RMetS
Royal Meteorological Society

Non-exponential return time distributions for vorticity extremes explained by fractional Poisson processes

R. Blender,[a*] C. C. Raible[b] and F. Lunkeit[a]
[a] *Dept. of Geosciences, Meteorologisches Institut, Universität Hamburg, Germany*
[b] *Climate and Environmental Physics and the Oeschger Centre for Climate Change Research, University of Bern, Switzerland*

*Correspondence to: R. Blender, University of Hamburg Meteorological Institute, Grindelberg 5, D-20144 Hamburg, Germany. E-mail: richard.blender@uni-hamburg.de

Serial correlation of extreme midlatitude cyclones observed at the storm track exits is explained by deviations from a Poisson process. To model these deviations, we apply fractional Poisson processes (FPPs) to extreme midlatitude cyclones, which are defined by the 850 hPa relative vorticity of the ERA interim reanalysis during boreal winter (DJF) and summer (JJA) seasons. Extremes are defined by a 99% quantile threshold in the grid-point time series. In general, FPPs are based on long-term memory and lead to non-exponential return time distributions. The return times are described by a Weibull distribution to approximate the Mittag–Leffler function in the FPPs. The Weibull shape parameter yields a dispersion parameter that agrees with results found for midlatitude cyclones. The memory of the FPP, which is determined by detrended fluctuation analysis, provides an independent estimate for the shape parameter. Thus, the analysis exhibits a concise framework of the deviation from Poisson statistics (by a dispersion parameter), non-exponential return times and memory (correlation) on the basis of a single parameter. The results have potential implications for the predictability of extreme cyclones.

Key Words: vorticity; extremes; cyclones; return times; fractional Poisson processes; long-term memory

Received 29 October 2013; Revised 18 February 2014; Accepted 18 February 2014; Published online in Wiley Online Library 29 March 2014

1. Introduction

Extremes such as midlatitude storms are a major source of natural hazards, as demonstrated impressively over the last two decades. Extreme storms tend to occur in families or trains, as the traditional view of synoptic cyclones suggests (Bjerknes and Solberg, 1922). A devastating example of such a family is storms *Lothar* and *Martin* during winter 1999/2000, which caused the highest level of damage of midlatitude cyclones up to now.

Apparently these events triggered strong scientific interest in analyzing the clustering of midlatitude cyclones in the Northern Hemisphere (Mailier *et al.*, 2006). Their results indicated that the return times of extreme midlatitude cyclones deviate from the assumed Poisson process, as the variance to mean ratio differs from one. At the beginning of the midlatitude storm tracks, a slight underestimation is found, while in the cores of the storm tracks and mainly in the lysis region the ratio is 50% higher than the Poisson process would suggest (Mailier *et al.*, 2006). Similarly, Franzke (2013) showed a long-term dependence and serial clustering of extreme wind speeds. Moreover, analyzing climate model simulations, serial clustering of midlatitude cyclones may change under future global warming (Pinto *et al.*, 2013). Thus, these studies concluded that the return time distribution deviates from an exponential decay and the authors assumed that the events are correlated, denoted as serial clustering. This assumption and the fact that the occurrence of extreme midlatitude cyclones cannot be modelled by a Poisson process point to a revisit of the model underlying extreme event occurrences.

In general, return time distributions of extreme event occurrences are assumed to be exponential. The stochastic model underlying extreme events is the Poisson process, which describes a rare event without correlation and a constant intensity. If extreme events are determined in time series with correlations, the return time distribution changes. However, the deviation from the exponential distribution is not necessarily an indicator of memory. Consider for example a distribution with two possible return times, a short time with high probability and a much larger time with small probability. Such a distribution yields clusters. The main ingredient is that intermediate return times are rare.

The first analyses proposed a stretched exponential distribution for long return times in time series with long-term memory (Bunde *et al.*, 2004). For short times, an independent power law was suggested. Later on, Blender *et al.* (2008) found for time series with long-term memory close to nonstationarity ($1/f$) that their return time distribution is accurately modelled by a Weibull distribution (WD). The WD combines both the short- and long-term limits. Furthermore, a model was provided by Santhanam and Kantz (2008). The Weibull distribution is widely used, for example as a limit of the extreme event distribution (GEV). Although the WD is a versatile distribution, physical models are still rare. A first success in this direction was provided by Jo *et al.*

(2011), who found the WD as the asymptotic form for a cascade process.

To revisit the model underlying extreme events such as cyclones, we investigate in this study the return time distribution of extremes in the relative vorticity fields in the lower troposphere at 850 hPa. The maxima (minima) of the relative vorticity are a direct measure for the position of cyclones in the Northern (Southern) Hemisphere and are used to define cyclones (Hodges, 1994). This is analogous to the standard method of detecting lows by minima in the geopotential height or mean sea-level pressure (e.g. Blender *et al.*, 1997; Raible *et al.*, 2008; Neu *et al.*, 2013, and the methods therein). Hence, we do not consider trajectories of cyclones (from the Lagrangian perspective) but extremes at grid points, representing the Eulerian point of view.

We apply fractional Poisson processes (FPPs: Laskin, 2003) as a model for correlated extreme events. FPPs are based on a waiting time distribution with long-term memory, the Mittag–Leffler function (ML), which is characterized by an additional parameter describing the deviation from an ordinary Poisson process. This function can be approximated by a Weibull distribution (WD) with a shape parameter corresponding to the parameter of the ML function. For a unity shape parameter, the relation is exact. A Weibull distribution (WD) is fitted to the return times of extremes of the relative vorticity at all grid points. The FPP model allows derivation of e.g. the variance to mean ratio used in Mailier *et al.* (2006) as a measure for the deviation from the Poisson behaviour. Hence, we are able to compare our analysis based on the Weibull distribution directly with the clustering found in Mailier *et al.* (2006). Furthermore, the deviation from a Poisson process can be explained by the memory of the time series.

This study is organized as follows. In section 2, FFPs are introduced and the main properties relevant for the present study are summarized. The analysis of the ERA interim data and the fit of the WD to extreme return times is presented in section 3. Based on the WD shape parameter, the dispersion is determined in section 4. An analysis of long-term memory yields an independent estimate of the shape parameter presented in section 5. The results are summarized and interpreted in section 6.

2. Fractional Poisson processes

A FPP is a stochastic process with long-term memory and a non-exponential waiting time distribution (Laskin, 2003). In the present application, the waiting times are identified as return times of extreme events. This model was suggested to describe the non-standard distributions found in a variety of complex systems. Mathematically, the FPP can be considered as a master equation with a waiting time distribution described by a fractional Poisson distribution. For the physical reasoning and the mathematical details, we refer the reader to Laskin (2003). The deviation from the standard Poisson process is described by a parameter μ, which is restricted to the range

$$0 < \mu \leq 1. \tag{1}$$

For $\mu = 1$, the FPP reduces to the Poisson process with an exponential waiting time distribution.

The mean of the FPP with intensity ν in the notation of Laskin (2003) is

$$\overline{n}_\mu = \frac{\nu t^\mu}{\Gamma(\mu+1)}, \tag{2}$$

with the Gamma function $\Gamma(n+1) = n!$.

The variance is

$$\sigma_\mu^2 = \overline{n}_\mu + \overline{n}_\mu^2 \left(\frac{\mu\Gamma(\mu)\Gamma(1/2)}{2^{2\mu-1}\Gamma(\mu+1/2)} - 1 \right). \tag{3}$$

The mean of events and their variance are determined by the intensity ν in a time interval t.

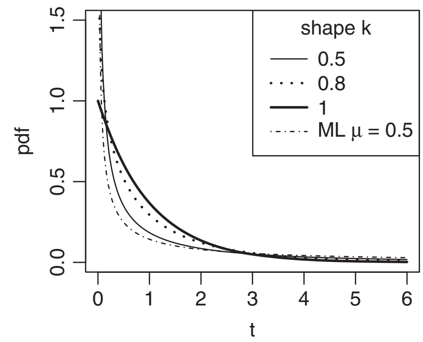

Figure 1. Weibull distribution for shape parameters $k = 0.5, 0.8, 1$ (scale $\lambda = 1$). The exact ML function for $\mu = 1/2$ (Eq. (6)) is included for comparison.

The waiting time distribution of the FPP is

$$\psi_\mu(\tau) = \nu\tau^{\mu-1}E_{\mu,\mu}(-\nu\tau^\mu), \tag{4}$$

where $E_{\mu,\mu}$ is the ML function, which becomes

$$E_{1,1}(x) = \mathrm{e}^x \tag{5}$$

in the limit $\mu = 1$ (for details see Laskin, 2003). For $\mu = 1/2$, the function is given by

$$E_{1/2,1/2}(x) = \mathrm{e}^{x^2}[1 + \mathrm{erf}(x)]. \tag{6}$$

In general, the ML function is defined by an integral representation or a corresponding series expansion.

In the present article, an approximation for the ML function in Eq. (4) is used. The function is simplified in such a way that in the limit $\mu = 1$ the standard Poisson process is recovered. We do not use asymptotic small or large time-scale expansions (Bianco *et al.*, 2007). The most simple approximation of the ML function is given by the limit $\mu = 1$ using Eq. (5). In this case $\psi_\mu(\tau)$ corresponds to a Weibull distribution (WD) with scale parameter λ and shape parameter k:

$$p(t, \lambda, k) = \frac{k}{\lambda}\left(\frac{t}{\lambda}\right)^{k-1} \mathrm{e}^{-(t/\lambda)^k}. \tag{7}$$

In Figure 1, the Weibull distribution is shown for the values $k = 0.5$, 0.8 and 1. The WD for $k = 0.5$ has to be compared with the distribution given by the ML function for $\mu = 1/2$. In the analysis below, the shape parameters are in the range 0.5–1, hence $\mu = 1/2$ is a lower limit. The diagram shows that the WD represents an acceptable approximation in this case. Compared with the exponential distribution ($k = 1$), the WD is skewed for $k < 1$, with higher frequencies for very small and very large times.

In the following, the FPP parameter μ is identified as the Weibull shape parameter k. Since the FPP is valid only in the range $0 < \mu \leq 1$ (Eq. (1)), care has to be taken whether the same applies to the Weibull shape parameter. In the WD there is no restriction for k, for example $k = 2$ is known as the Rayleigh distribution.

A few artificial snapshots for events with return time distributions for different shape parameters k are presented in Figure 2 (the scale parameter is unity in all examples). For the

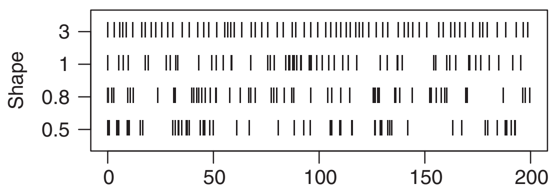

Figure 2. Events with return times following a Weibull distribution for different shape parameters k (scale $\lambda = 1$).

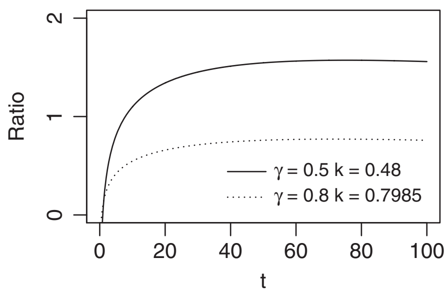

Figure 3. Ratio of stretched exponential and Weibull distribution for exponents $\gamma = 0.5,\ 0.8$ in Eq. (9) and shape parameters $k = 0.48,\ 0.7085$ in Eq. (7), as indicated (scales $T = \lambda = 1$).

values $k = 0.5,\ 0.8$ there is enhanced clustering compared with $k = 1$, representing a standard Poisson process. The reason is the skewness of the WD (see Figure 1). Note that there is no correlation between succeeding return times. In the results below, parameters in range $k \approx 0.5$ up to $k \approx 0.9$ are found. For the higher shape parameter $k = 3$, which is included for comparison only, a quasi-regular behaviour is seen.

An essential motivation for the use of the Weibull distribution is its relevance for long-term correlated data, where the correlation function decays according to a power law

$$C(t) \sim t^{-\gamma}, \tag{8}$$

with the correlation exponent γ. Blender *et al.* (2008) and Santhanam and Kantz (2008) found that for such data the return time distribution is given by a WD (Eq. (7)). The WD combines a power law and the stretched exponential

$$p_s(t) \sim e^{-(t/T)^{\gamma}}, \tag{9}$$

which had been suggested earlier as a distribution for return times in power-law correlated data (Bunde *et al.*, 2003). The WD is, in particular, preferable for time series with strong memory with γ close to zero, i.e. $1/f$ noise (Blender *et al.*, 2008).

The stretched exponential distribution can be well approximated by a WD with $k \approx \gamma$ for long time-scales. To demonstrate this, Figure 3 shows the ratios of a stretched exponential distribution and a WD for $\gamma = 0.5, k = 0.48$ and $\gamma = 0.8, k = 0.7985$, respectively. The absolute values are not considered here, since the short-term behaviour is not included (necessary for normalization).

Therefore, we identify the Weibull shape parameter k as the correlation exponent γ in Eq. (8):

$$k \approx \gamma. \tag{10}$$

This relationship is a basis for the present analysis, since it allows derivation of an independent measure for the deviation from standard Poisson processes by a memory analysis.

The Weibull distribution can be considered as an extension of a physical decay time-scale analysis. The additional shape parameter describes a skewness of the exponential decay. Since the WD is motivated by physical reasoning, we do not try to apply significance tests to find whether another two-parameter distribution is more apt than WD. An advantage of the WD is that it is a versatile and well-known distribution. The parameters and their uncertainty are estimated by a Maximum Likelihood Estimation (MLE) fit using the statistical package 'fitdistr' of R (R Core Team, 2012).

3. Return time distribution

The data analyzed in this study are ERA interim reanalyses with a resolution of 1.5° during boreal winter (DJF) and summer (JJA) seasons. The return times of cyclonic events are determined in relative vorticity on the 850 hPa pressure level, including all 6 h time steps. The time period ranges from 1 January 1980 to 28 February 2013. The first winter consists of Jan–Feb data only. The standard deviation of the vorticity is concentrated in the storm tracks of the Northern and Southern Hemispheres during boreal (DJF) and austral winter (JJA; Figure 4).

The extremes are determined as exceedances of the 99% quantile, which is adjusted separately for each grid point. Thus, the method is a peak-over-threshold (POT) and respects differences between regions. In the Southern Hemisphere, minima of the relative vorticity are considered. Return times are determined for all grid points and a Weibull distribution (WD) is fitted to determine the scale and shape parameters λ and k by an MLE algorithm. There are marked differences in scale between the polar regions, the midlatitudes and the Tropics (Figure 5). The largest scales appear clearly in both midlatitudes, with higher values in the SH. The uncertainty in the scales given by the MLE fit is of the order of 25%.

A first noteworthy outcome is that all shape parameters (Figure 6) detected are below one, hence the requirement that Eq. (1) is also satisfied for the Weibull shape parameter. The uncertainty in the shape parameters is ≈ 0.04 (by MLE). Thus the data do not contradict the identification of $k = \mu$. This result allows the interpretation of the extreme events in the vorticity as a fractional Poisson process. The high values $k \approx 0.7$–0.9 in the genesis regions of the midlatitude storm tracks are closest to a Poisson process. If deviations from this are interpreted as correlations, these areas have the weakest memory. Low values appear in the Tropics, where $k \approx 0.5$.

Using lower quantiles in estimating extreme 850 hPa relative vorticity leads to changes in the shape parameter (not shown). Low values (≈ 0.4) increase while high values (≈ 0.8) decrease; hence the shape parameters tend to the mean.

4. Dispersion

To describe the deviation of cyclone occurrence from a standard Poisson process, Mailier *et al.* (2006) use the dispersion

$$\Psi = \frac{\sigma_n^2}{\bar{n}} - 1. \tag{11}$$

The dispersion was determined by the variances σ_n^2 and the means $\bar{n}$ of cyclone passages on a monthly scale. In a Poisson process, the parameter vanishes due to the equality of the mean and the variance. For $\Psi \approx 0$, cyclones occur according to a Poisson process (see Figure 2 for the shape parameter $k = 1$). Positive Ψ are interpreted as serial clustering. For example, for cyclones in the exit regions of the North Atlantic storm tracks, values of 0.5 are found (indicated by a percentage of 50 in Mailier *et al.*, 2006).

With the mean and variance of FPPs in Eqs (2) and (3), the dispersion parameter can be expanded for small deviations $1 - \mu$:

$$\Psi \approx \nu t(1 - \mu). \tag{12}$$

The leading order of the dispersion is linear in the parameter μ and vanishes for $\mu = 1$, the exponential distribution. The dispersion depends on the mean νt and hence on the time-scale t, due to the squared mean in the FPP variance (Eq. (3)). The mean of the extreme events is determined by the 99% quantile threshold and the duration t; hence, for 1 month (as in Mailier *et al.*, 2006), $\nu t \approx 1.2$. Due to the fixed quantile threshold, this mean is constant in our analysis. Thus the shape parameter $k = \mu$ can be used for a comparison of serial correlations of Lagrangian cyclones in Mailier *et al.* (2006).

Here we present results for the factor $\Psi/\nu t = 1 - k$ to estimate the dispersion based on the shape parameter (Figure 7). In the genesis regions, the lowest values, $1 - k \approx 0.2$–0.3, are found. Mailier *et al.* (2006) present values close to zero. Cyclogenesis behaves nearly like an uncorrelated Poisson process (rare events) with an exponential return time distribution.

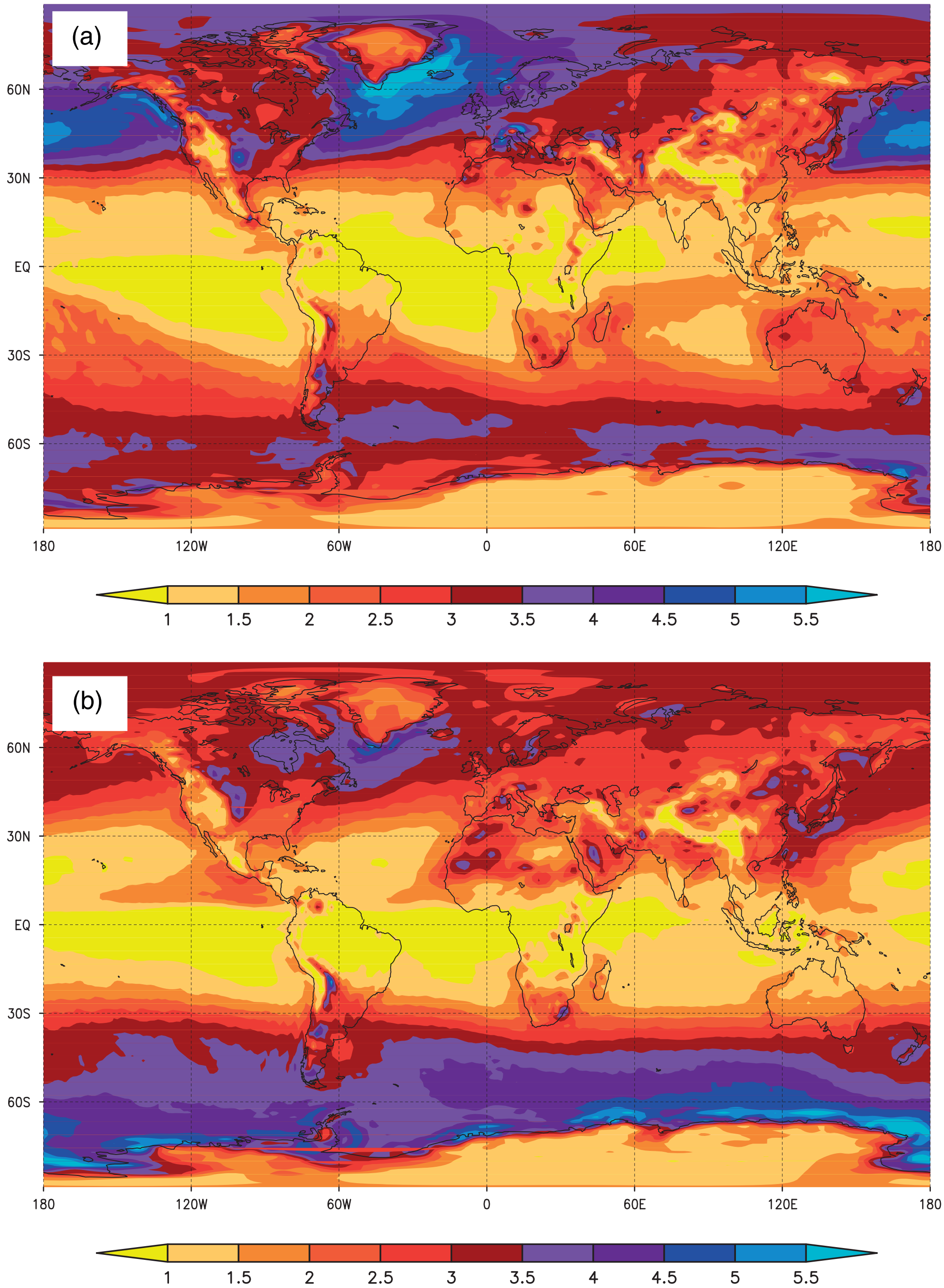

Figure 4. Standard deviation of the vorticity in (a) DJF and (b) JJA ($10^{-5}\,s^{-1}$).

The dispersion increases at the exits of the storm tracks, with $\Psi/\nu t = 1 - k \approx 0.4$ (for example close to Europe). This has to be compared with $\Psi \approx 0.5$ in Mailier *et al.* (2006). In these regions, non-exponential return time distributions and serial correlations are identified.

During summer, the dispersion indicates serial correlation throughout the storm tracks of the Northern Hemisphere (Figure 7(a)). Note that the extremes occur on shorter time-scales, as indicated by λ (Figure 5(a)).

The results suggest a tentative identification of cyclogenetic regions on the basis of an analysis of the return time distribution. If genesis regions of storms are identified by low values of dispersion (as in the Northern Hemispheric winter: Mailier *et al.*, 2006), sources of cyclones can be identified in the boreal summer (Figure 7(a)). Examples with maxima are the Northern American continent (the Great Plains) and tropical East and West Africa. Noteworthy is a maximum in the western (Spain) but not in the eastern Mediterranean.

The Southern Hemisphere during austral winter (JJA) reveals a distinct non homogenous pattern of dispersion (Figure 7). An outstanding cyclogenetic region can be identified at 30°E–150°E, 40°S–50°S, as in Simmonds and Keay (2000).

5. Memory

Fractional Poisson processes are associated with long-term memory. Long-term memory is a theoretical concept, which is limited in observations to finite time intervals. Here a lower limit is given by the upper time-scale of synoptic eddies, for which 5 days is chosen. An upper limit is chosen as 30 days, to remain below the seasonal time-scale.

Short-term memory has a finite time-scale and is described by an exponential decay. In the present data, such a decay is found roughly up to 5 days. The events for $k = 0.5$ in Figure 2 can be used to visualize both time-scales: the duration of a

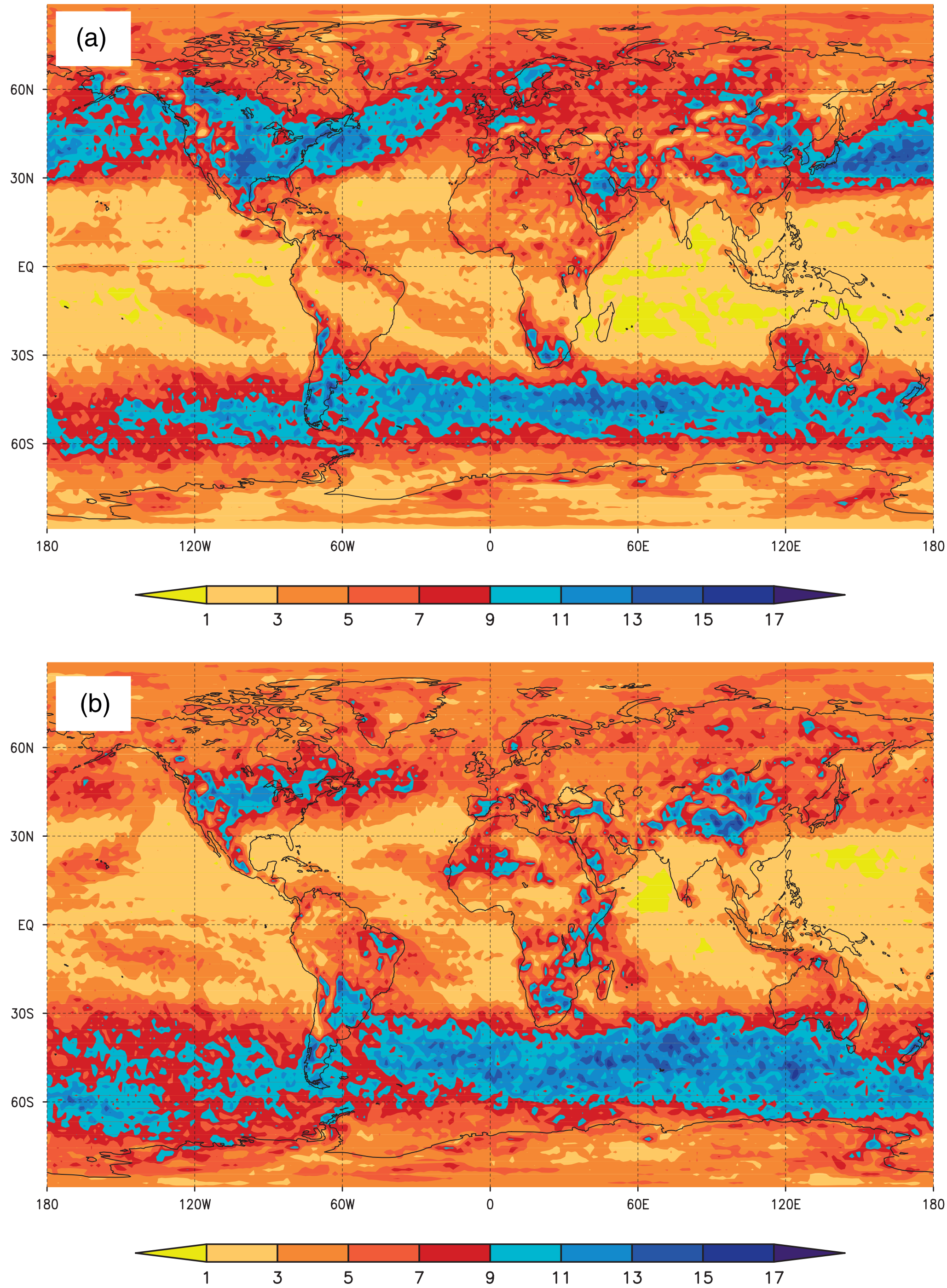

Figure 5. Weibull scale parameter in (a) DJF and (b) JJA (days).

cluster corresponds to the lower limit of 5 days, while the cluster separation is given by the upper limit of 30 days.

Long-term memory has no time-scale and the correlation function is not integrable (Fraedrich and Blender, 2003). Mathematically this is formulated by a power-law decay of the correlation function with a correlation exponent γ (Eq. (8)). The decay of the correlation function is related to the scaling of the power spectrum, $S(f) \sim f^{-\beta}$ with frequencies f, by the relationship $\beta = 1 - \gamma$ for the spectral exponent β.

A method to retrieve long-term memory in stationary time series is given by the detrended fluctuation analysis (DFA: Peng *et al.*, 1994; Fraedrich and Blender, 2003). This method determines the fluctuation function, $F(t) \sim t^{\alpha}$, with the fluctuation exponent α; the three exponents are related by $\beta = 2\alpha - 1 = 1 - \gamma$.

In the present analysis, the fluctuation function shows a power law beyond the short term memory, hence beyond several days. A power law has also been observed by Tsonis *et al.* (1999) for the 500 hPa geopotential height variability. An upper limit cannot be determined, since the seasonal scale provides a break. Therefore, the scaling exponent α is fitted in the time interval 5–30 days. Values of the exponent β are in the range 0–0.5 (not shown). Small exponents are found in the midlatitudes, while $\beta \approx 0.5$ dominates the Tropics.

Based on the identification of Eq. (10), it is possible to estimate the shape parameter k of the WD by a memory analysis, $k_{\mathrm{LTM}} = \gamma = 2 - 2\alpha$ (Figure 8). The shape parameter k_{LTM} shows low values below 0.4 in a few areas, an intermediate range 0.4–0.8 in the largest part of the Tropics and a high range 0.8–1 in the midlatitudes. The highest values are found close to the genesis regions of the cyclone tracks in the Northern Hemisphere during DJF and in the Southern Hemispheric storm track during JJA. A large area with high k_{LTM} on the North American continent during winter is outstanding. The physical reason is low memory in the time series ($\beta \approx 0$, or white noise). The exit regions of the midlatitude storm tracks in the Northern Hemisphere are dominated by low and intermediate values of the shape parameter k_{LTM}, highlighting the existence of long-term memory and thus the potential of increased predictability in these regions.

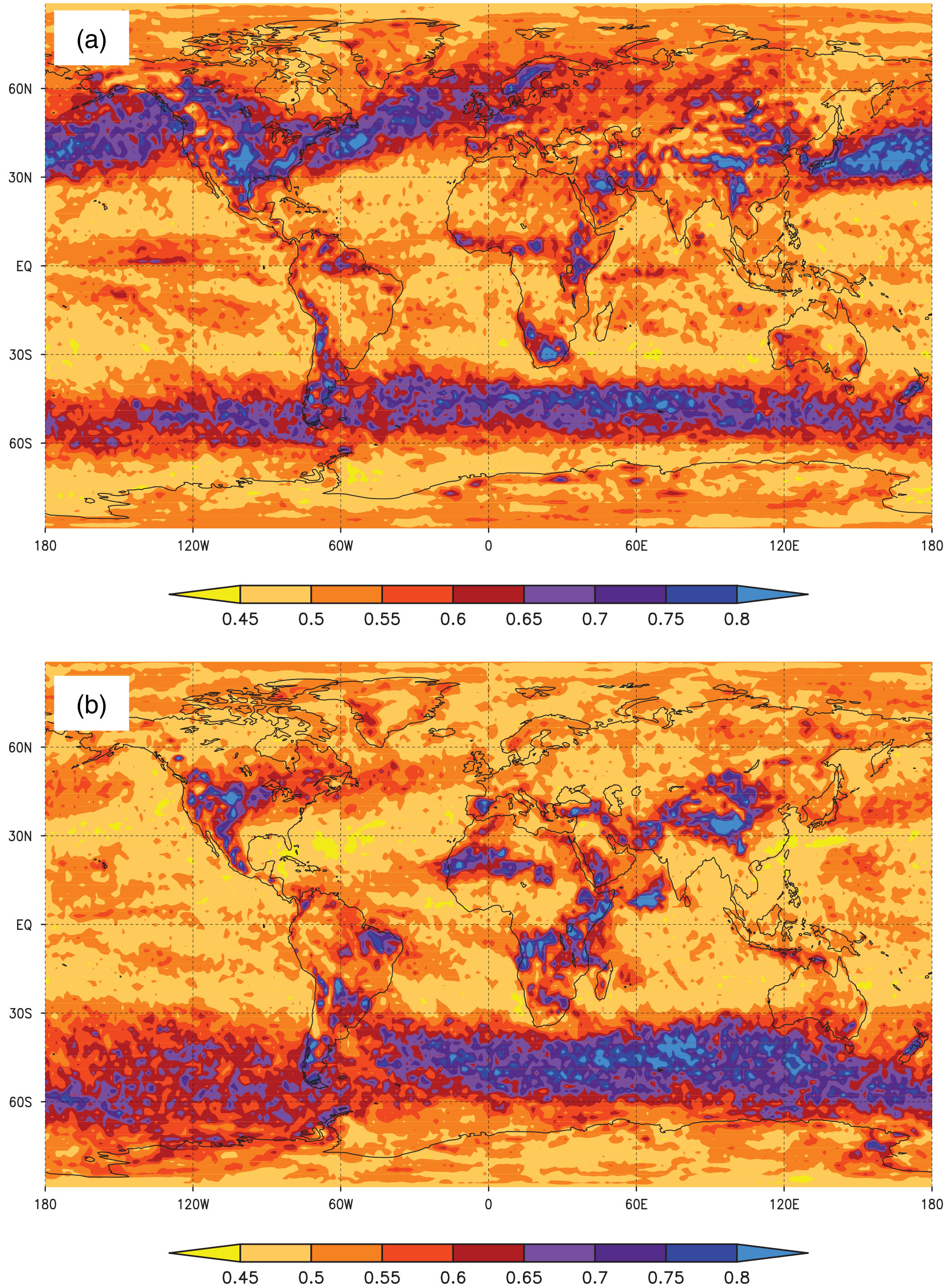

Figure 6. Weibull shape parameter k in (a) DJF and (b) JJA.

The global behaviour follows the Weibull shape parameter k (Figure 6). Clearly, the agreement is limited, since several approximations enter this analysis, for example the use of a threshold and the fit of the fluctuation function in a finite time interval (while an ideal fractional Poisson process assumes long-term memory on all time-scales). Furthermore, the approximation of the stretched exponential parameter with the Weibull shape parameter is coarse, with an uncertainty of ≈ 0.1. Nevertheless, there is sufficient correspondence supporting the hypothesis that the memory in fractional Poisson processes is relevant for the Weibull shape parameter and the dispersion in the recurrence of extreme vorticity events.

6. Summary and conclusions

The aim of the present study is to assess serial correlations in midlatitude storms. The high relevance of such correlations is given by the damage caused if the natural environment or social institutions do not have time to recover. Serial correlations have been found previously for midlatitude cyclones in the Northen Hemispheric winter. The findings are characterized in terms of a dispersion parameter, which is defined as the deviation from the equality of variance and means characterizing Poisson processes. In the presence of serial correlations, the assumption of exponentially distributed return times is not valid.

In the present study, extremes of vorticity are analyzed in an Eulerian framework. We use return times of extremes in vorticity in ERA interim reanalysis data at 1.5° resolution from 1980–2013. The analysis encompasses winter and summer seasons and considers the Northern and Southern Hemispheres. An advantage is that the amount of available data for analysis is substantially larger than for cyclone trajectories. The results show that the Eulerian analysis is also relevant for midlatitude cyclones.

The new step is to consider extreme events as fractional Poisson processes. These processes are based on long-term memory and yield a non-exponential return time distribution given by the ML function. The deviation from standard Poisson processes is

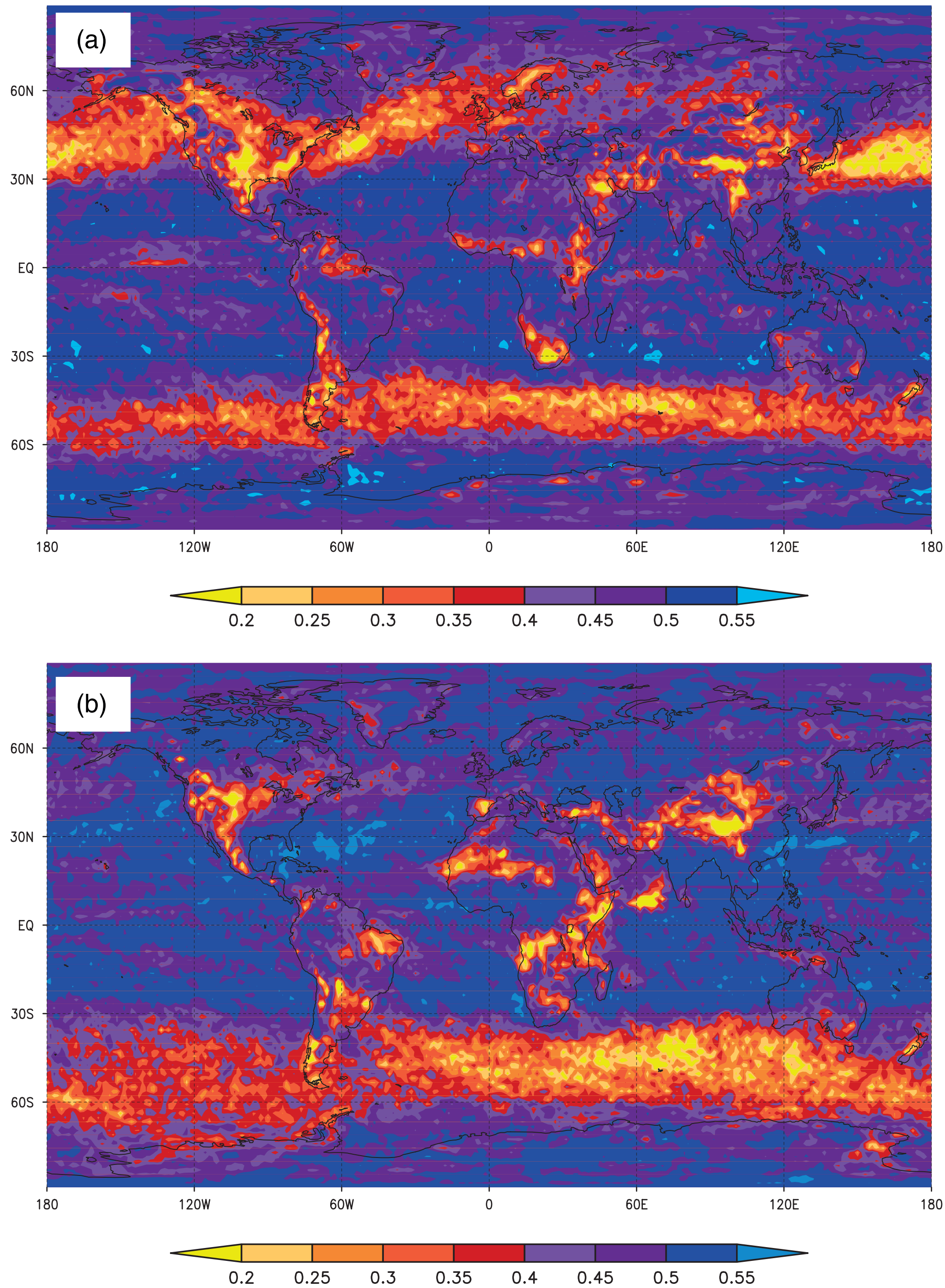

Figure 7. Dispersion $\Psi/\nu t = 1 - k$ based on the Weibull shape parameter k in (a) DJF and (b) JJA.

described by a single parameter μ, which becomes $\mu = 1$ for the exponential distribution.

The ML function is approximated by the Weibull distribution (WD). The reason for this is twofold: firstly, the WD is a versatile distribution with a wide range of applications. The fit is easily performed in available software routines (here R) using the standard MLE approach. A second and equally important reason is that the WD describes the return time statistics in time series with long-term memory.

The shape parameters are below one, which is a condition for interpretation as the FPP parameter. The scale and the shape parameters distinguish between the Tropics and midlatitudes, but also the entrance and exit regions of midlatitude storms.

In the cyclogenetic regions of midlatitude storm tracks, the shape parameter is highest (≈ 0.8), indicating uncorrelated cyclone development. In the exit regions of the storm tracks, the shape parameter reduces to about 0.5, indicating that the extremes are serially correlated. The absence of correlation in the cyclogenetic regions suggests an identification of these regions by a pure statistical analysis.

The shape parameter determines the dispersion index, which has been used to identify the deviations from standard exponential return times and an underlying Poisson process. The results confirm results for the dispersion determined by cyclones almost quantitatively. Thus, the results reconcile the Eulerian and Lagrangian approaches.

A main finding is that the memory in the vorticity time series determines the shape parameter of the WD. A unified view of correlated extremes based on different concepts emerges:

- return time distribution;
- dispersion; and
- memory.

All three aspects use the Weibull shape parameter as a single identifier. The three measures are equivalent characterizations of fractional Poisson processes.

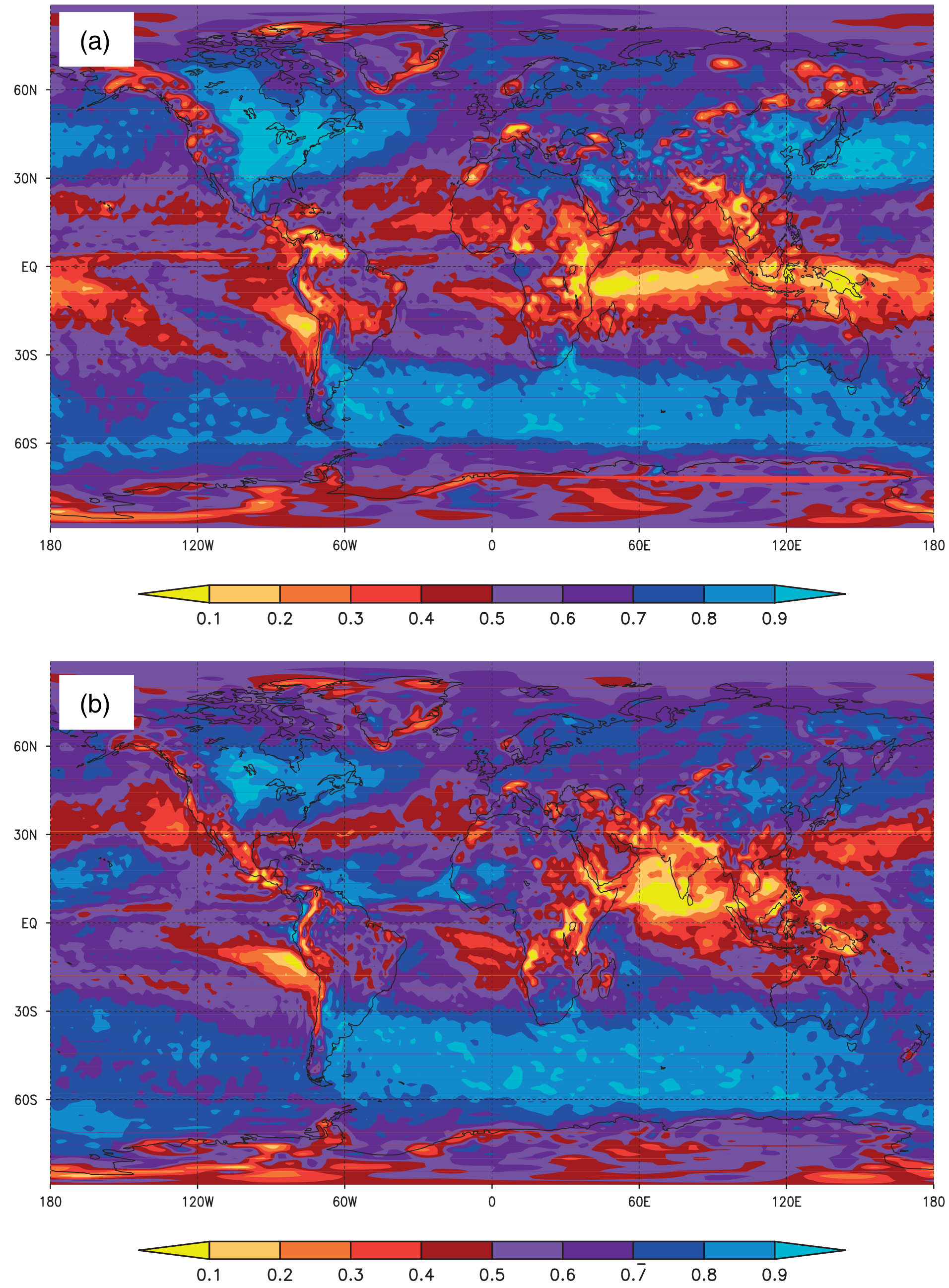

Figure 8. Shape parameter predicted by long-term memory analysis, $k_{\mathrm{LTM}} = 1 - \beta$, where β is the spectral exponent for (a) DJF and (b) JJA.

The serial correlation in the vorticity time series is transferred to the correlation of extremes and allows predictability. For power-law correlated data (Eq. (8)), Bunde *et al.* (2004) found correlations of successive return times with a similar power law. Blender *et al.* (2008) observed that the correlation of the return times is distinctly lower than the original time series in data with strong memory, corresponding to $k = 0$ (close to nonstationarity).

Acknowledgements

We thank Cristian Franzke for discussions. We are grateful for many discussions with the members of the IMILAST project (intercomparison of midlatitude storm diagnostics, contact Urs Neu, Bern).

References

Bianco S, Ignaccolo M, Rider MS, Ross MJ, Winsor P, Grigolini P. 2007. Brain, music, and non-Poisson renewal processes. *Phys. Rev. E* **75**: 061911, doi: 10.1103/PhysRevE.75.061911.

Bjerknes J, Solberg H. 1922. Life cycle of cyclones and the polar front theory of atmospheric circulation. *Geofys. Publ.* **3**: 3–18.

Blender R, Fraedrich K, Lunkeit F. 1997. Identification of cyclone-track regimes in the North Atlantic. *Q. J. R. Meteorol. Soc.* **123**: 727–741, doi: 10.1002/qj.49712353910.

Blender R, Fraedrich K, Sienz F. 2008. Extreme event return times in long-term memory processes near $1/f$. *Nonlinear Processes Geophys.* **15**: 557–565.

Bunde A, Eichner JF, Havlin S, Kantelhardt JW. 2003. The effect of long-term correlations on the return periods of rare events. *Physica A* **330**: 1–7, doi: 10.1016/j.physa.2003.08.004.

Bunde A, Eichner JF, Havlin S, Kantelhardt JW. 2004. Return intervals of rare events in records with long-term persistence. *Physica A* **342**: 308–314, doi: 10.1016/j.physa.2004.01.069.

Fraedrich K, Blender R. 2003. Scaling of atmosphere and ocean temperature correlations in observations and climate models. *Phys. Rev. Lett.* **90**: 108501, doi: 10.1103/PhysRevLett.90.108501.

Franzke CLE. 2013. Persistent regimes and extreme events of the North Atlantic atmospheric circulation. *Philos. Trans. R. Soc. A* **371**: 20120518, doi: 10.1098/rsta.2011.0471.

Hodges KI. 1994. A general method for tracking analysis and its application to meteorological data. *Mon. Weather Rev.* **122**: 2573–2586.

Jo J, Fortin JY, Choi MY. 2011. Weibull-type limiting distribution for replicative systems. *Phys. Rev. E* **83**: 031123, doi: 10.1103/PhysRevE.83.031123.

Laskin N. 2003. Fractional Poisson process. *Commun. Nonlinear Sci. Numer. Simul.* **8**: 201–213.

Mailier PJ, Stephenson DB, Ferro CAT, Hodges KI. 2006. Serial clustering of extratropical cyclones. *Mon. Weather Rev.* **134**: 2224–2240, doi: 10.1175/MWR3160.1.

Neu U, Akperov MG, Bellenbaum N, Benestad RS, Blender R, Caballero R, Cocozza A, Dacre HF, Feng Y, Fraedrich K, Grieger J, Gulev S, Hanley J, Hewson T, Inatsu M, Keay K, Kew SF, Kindem I, Leckebusch GC, Liberato MLR, Lionello P, Mokhov II, Pinto JG, Raible CC, Reale M, Rudeva I, Schuster M, Simmonds I, Sinclair M, Sprenger M, Tilinina ND, Trigo IF, Ulbrich S, Ulbrich U, Wang XL, Wernli H. 2013. IMILAST a community effort to intercompare extratropical cyclone detection and tracking algorithms. *Bull. Am. Meteorol. Soc.* **94**: 529–547, doi: 10.1175/BAMS-D-11-00154.1.

Peng CK, Buldyrev SV, Havlin S, Simons M, Stanley HE, Goldberger AL. 1994. Mosaic organization of DNA nucleotides. *Phys. Rev. E* **49**: 1685–1689, doi: 10.1103/PhysRevE.49.1685.

Pinto JG, Bellenbaum N, Karremann MK, Della-Marta PM. 2013. Serial clustering of extratropical cyclones over the North Atlantic and Europe under recent and future climate conditions. *J. Geophys. Res.* **118**: 12476–12485, doi: 10.1002/2013JD020564.

Raible CC, Della-Marta P, Schwierz C, Wernli H, Blender R. 2008. Northern Hemisphere extratropical cyclones: A comparison of detection and tracking methods and different reanalyses. *Mon. Weather Rev.* **136**: 880–897, doi: 10.1175/2007MWR2143.1.

R Core Team. 2012. '*R: A language and environment for statistical computing*', Technical Report. R Foundation for Statistical Computing: Vienna, Austria. http://www.r-project.org/.

Santhanam MS, Kantz H. 2008. Return interval distribution of extreme events and long-term memory. *Phys. Rev. E* **78**: 051113, doi: 10.1103/PhysRevE.78.051113.

Simmonds I, Keay K. 2000. Mean southern hemisphere extratropical cyclone behaviour in the 40-year NCEP-NCAR reanalysis. *J. Clim.* **13**: 873–885.

