1 Analytical representation of the pulse response functions

1.1 HRBM/CTBM

A pulse like carbon input into the assimilation pools has been forced by increasing the atmospheric CO_2 level by 2 ppmv above the pre-industrial equilibrium; after one year CO_2 is set back again to the pre-industrial value. The perturbation in the heterotrophic respiration was then monitored for 500 years. The perturbative flux was fitted by analytical functions of age t. Different functions are used for different time intervals in order to assure a precise representation. Finally, the decay response function has been normalized in such a way that its integral from time 0 to infinity yields 1 to obtain the following expressions where R_{decay} is the decay response function in units of 1/year and t is age measured in years:

$$R_{decay}(t=1) : 0.05113475$$

$$R_{decay}(t=2) : 0.17441086$$

$$\begin{array}{rcl}
\text{For } t \in [2,14] : & & \\
R_{decay}(t) & = & 0.63052300 \cdot 10^{-2} \\
& + & 0.34542444 \cdot 10^{1} \cdot \exp(-t/0.90420370) \\
& + & 0.14162310 \quad \cdot \exp(-t/2.6164904) \\
& - & 0.50115801 \cdot 10^{1} \cdot \exp(-t/0.68991461) \\
\text{check value} & : & R_{decay}(t=14) : 0.69778697 \cdot 10^{-2} \\
\text{For } t \in [15,450] : & & \\
R_{decay}(t) & = & 0.47219194 \cdot 10^{-4} \\
& - & 0.24442864 \cdot 10^{1} \cdot \exp(-t/86.270923) \\
& + & 0.24497386 \cdot 10^{1} \cdot \exp(-t/86.2692922) \\
& + & 0.54808383 \cdot 10^{-2} \cdot \exp(-t/13.9898764) \\
\text{check value} & : & R_{decay}(t=450) : 0.75504533 \cdot 10^{-4} \\
\text{For } t \geq 451 : & & \\
R_{decay}(t) & = & 0.20304167 \cdot 10^{2} \cdot \exp(-t/45.166515) \\
& - & 0.23764954 \cdot 10^{2} \cdot \exp(-t/44.364017) \\
& + & 0.15264106 \cdot 10^{-3} \cdot \exp(-t/438.32634) \\
\text{check value} & : & R_{decay}(t=500) : 0.62047155 \cdot 10^{-4}
\end{array}$$

1.2 FBM

The response function of the FBM has been determined by minimizing the difference in the heterotrophic plus autotrophic respiration flux as obtained by the full FBM and the pulse substitute model for scenarion IPCC WRE 1000. The analytical productivity function for the FBM is applied in the substitute. Then, the following expression for the decay response function r_{decay} was obtained where t is again the age in units of years and r_{decay} is in 1/year:

$$\begin{aligned}
R_{decay}(t=1) &= 0.391000556 \\
R_{decay}(t) &= 0.162881841 \cdot 10^{-1} \cdot \exp(-t/7.2055005) \\
&+ 0.339253611 \cdot 10^{-1} \cdot \exp(-t/14.732705) \\
&+ 0.974393294 \cdot 10^{-4} \cdot \exp(-t/29.538920) \\
&+ 0.513096896 \cdot 10^{-3} \cdot \exp(-t/117.50538) \\
\text{check value} &: R_{decay}(t=20) : 0.10226091 \cdot 10^{-1}
\end{aligned}$$

1.3 4-box Biosphere

The response of the 4box biosphere has been obtained analytically by solving the model equations for a perturbation in the the biosphere to atmosphere flux forced by an initial carbon input into the assimilation reservoirs. The response ($r_{decay}(t)$) is:

$$\begin{aligned}
r_{decay}(t) &= 0.70211 \cdot \exp(-0.35 \cdot t) + 0.01341 \cdot \exp(-0.05 \cdot t) \\
&- 0.7185 \cdot \exp(-0.458333 \cdot t) + 0.002932 \cdot \exp(-0.01 \cdot t),
\end{aligned}$$

where t is in units of years and $r_{decay}(t)$ in 1/year.

1.4 Box Substitute of HRBM/CTBM

The response of the Box Substitute Model of the HRBM/CTBM has been obtained analytically by solving the model equations for a perturbation in the the biosphere to atmosphere flux forced by an initial carbon input into the assimilation reservoirs (Equation 11 in the manuscript).

The response ($r_{decay}(t)$) is:

$$\begin{aligned}
r_{decay}(t) &= 0.3807325933 \cdot 10^{+0} \cdot \exp(-0.6777865291 \cdot 10^{+0} \cdot t) \\
&- 0.7674658037 \cdot 10^{+0} \cdot \exp(-0.4973317623 \cdot 10^{+1} \cdot t) \\
&+ 0.8421987407 \cdot 10^{-2} \cdot \exp(-0.1124884784 \cdot 10^{+0} \cdot t) \\
&+ 0.5582601879 \cdot 10^{-2} \cdot \exp(-0.1349562965 \cdot 10^{-1} \cdot t) \\
&+ 0.4099895526 \cdot 10^{-3} \cdot \exp(-0.3940026741 \cdot 10^{-2} \cdot t)
\end{aligned}$$

where t is in units of years and $r_{decay}(t)$ in 1/year.