1 Analytical representation of the productivity functions

1.1 HRBM/CTBM

The full HRBM/CTBM has been forced by increasing atmospheric CO2 in the model at every year stepwise by 2 ppm between 260 and 1274 ppmv. Total net primary productivity is recorded for each CO_2 level and the data were fitted by the following polynomial expression:

$$
\begin{aligned}
f_{prod} &= -3.936199 \cdot 10^{+01} & CO_2 \geq 260 ppmv \\
&+ 6.499772 \cdot 10^{-01} \cdot CO_2 - 2.142978 \cdot 10^{-03} \cdot (CO_2)^2 \\
&+ 4.312995 \cdot 10^{-06} \cdot (CO_2)^3 - 5.542613 \cdot 10^{-09} \cdot (CO_2)^4 \\
&+ 4.251490 \cdot 10^{-12} \cdot (CO_2)^5 - 1.247683 \cdot 10^{-15} \cdot (CO_2)^6 \\
&- 8.983608 \cdot 10^{-19} \cdot (CO_2)^7 + 1.093244 \cdot 10^{-21} \cdot (CO_2)^8 \\
&- 4.520093 \cdot 10^{-25} \cdot (CO_2)^9 + 7.076525 \cdot 10^{-29} \cdot (CO_2)^{10}
\end{aligned}
$$

where f_{prod} is the total NPP in GtC/yr and CO_2 is the atmospheric concentration in ppmv. The expression is valid for CO_2 concentrations larger than 260 ppmv. The pre-industrial NPP, $f_{prod,0}$ is 41.6713541 GtC/yr for a CO_2 level of 277.95 ppmv (check value).

1.2 FBM

The full FBM has been forced by prescribing atmospheric CO_2 according to the IPCC scenario WRE1000 and both CO_2 and gross primary productivity have been recorded. The following analytical expression again obtained by least square optimization is valid for CO_2 values between 278 and 1000 ppmv:

$$
f_{prod} = \frac{a_0 + a_1 \cdot CO_2 + a_2 \cdot CO_2^2 + a_1 \cdot CO_2^3}{b_0 + b_1 \cdot CO_2 + b_2 \cdot CO_2^2 + b_1 \cdot CO_2^3} \qquad CO_2 \in [278, 1000]
$$

$$
\begin{aligned}
a_0 &= 0.546877302 \cdot 10^4 & a_1 &= -0.337974427 \cdot 10^6 \\
a_2 &= -0.152891489 \cdot 10^9 & a_3 &= -0.750006896 \cdot 10^{10} \\
b_0 &= 0.237313645 \cdot 10^2 & b_1 &= 0.151601409 \cdot 10^4 \\
b_2 &= 0.232817871 \cdot 10^6 & b_3 &= -0.358729498 \cdot 10^9
\end{aligned}
$$

f_{prod} is the total gross primary productivity in GtC/yr and CO_2 is in ppmv. The pre-industrial GPP, $f_{prod,0}$ is 124.2157243 GtC/yr for a CO_2 level of 277.95 ppmv (check value).

1.3 4box Biosphere

The perturbative net primary productivity of the 4-box biosphere (δf_{prod}) is assumed to be a logarithmic function of the atmospheric CO_2

$$\delta f_{prod} = f_{prod}(t_0) \cdot \beta \cdot ln(\frac{CO_{2,a}(t)}{CO_{2,a}(t_0)}),$$

where the preindustrial productivity $f_{prod,0}$ is 60 Gt C/yr, the scaling factor β is 0.278 and the preindustrial CO_2 value is 278 ppm.

1.4 Box Substitute Model of HRBM/CTBM

In the Box Substitute Model of the HRBM/CTBM, Net Primary Productivity depends on CO_2 and Temperature ($f_{prod} = f_1(CO_2) \cdot f_2(Temp)$). The function f_1 is the same as for the Puls response Substitute of HRBM/CTBM:

$$\begin{aligned}
f_1 &= -3.936199 \cdot 10^{+01} \qquad\qquad CO_2 \geq 260ppmv \\
&+ 6.499772 \cdot 10^{-01} \cdot CO_2 - 2.142978 \cdot 10^{-03} \cdot (CO_2)^2 \\
&+ 4.312995 \cdot 10^{-06} \cdot (CO_2)^3 - 5.542613 \cdot 10^{-09} \cdot (CO_2)^4 \\
&+ 4.251490 \cdot 10^{-12} \cdot (CO_2)^5 - 1.247683 \cdot 10^{-15} \cdot (CO_2)^6 \\
&- 8.983608 \cdot 10^{-19} \cdot (CO_2)^7 + 1.093244 \cdot 10^{-21} \cdot (CO_2)^8 \\
&- 4.520093 \cdot 10^{-25} \cdot (CO_2)^9 + 7.076525 \cdot 10^{-29} \cdot (CO_2)^{10}
\end{aligned}$$

The best fit of the modification of Net Primary Productivity due to Temperature was obtained by the following function:

$$\begin{aligned}
f_2 &= 0.99922770 \cdot 10^{+0} \\
&+ 0.11780208 \cdot 10^{+0} \cdot \tanh(T/(0.509312421 \cdot 10^{+2})) \\
&+ 0.24305130 \cdot 10^{-2} \cdot \tanh(T/(0.885326739 \cdot 10^{+1}))
\end{aligned}$$

where f_{prod} is the total NPP in GtC/yr and CO_2 is the atmospheric concentration in ppmv, and T is the deviation from global equilibrium Temperature in °C.The expression is valid for CO_2 concentrations between 265 ppmv and 1400 ppmv and Temperature deviations between 0 and 5 °C.