Correcting the carbon cycle representation: How important is it for the economics of climate change?

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Economic analyses of the greenhouse effect are typically carried out within the framework of computable general equilibrium models which represent the climate system by simple two box proxies based upon the pioneering work of Nordhaus. Since errors in predicting the carbon budget can imply high costs, there is some need to include more sophisticated climate models into the economics of global climate change. This paper presents a non-linear pulse representation of the process-based and data-validated Bern carbon model. Compared to the Nordhaus approach this leads to different results with respect to optimal climate policy and atmospheric carbon dioxide concentration. In particular, our results suggest that economic studies which use a Nordhaus representation of the climate system are biased towards high carbon emission and low abatement levels.

Keywords: climate modeling, abatement policy, discounting, carbon cycle, non-linear pulse response representation

1. Introduction

In a recent *Nature* article Schimel [10] argues that "there is a pressing need to find out more about the relationship between anthropogenic emissions of the main greenhouse gas, CO_2, and the resulting atmospheric concentration"; in particular, since "errors in predicting the carbon budget could come at high economic costs". However, from an economist's point of view such an effort can be recommended only if it must be expected that investing in more sophisticated carbon cycle models will generate new insight into the economics of global warming or can lead to different policy recommendations.

Today, sophisticated models are available to simulate the dynamics of the atmosphere–ocean–biosphere system for understanding the fate of carbon emissions and the response of the climate system to greenhouse gas forcing. But the complexities and computational demands of these models prevent their direct application in integrated assessments. As such, there is a need not only to improve existing carbon cycle models. There is also a need to develop proxies of highly complex and data demanding climate models that can transfer the climate scientists' knowledge into the economic analysis of the greenhouse effect and allow for computational experiments under the existing restrictions in computing capacities and time.

Nordhaus [7] has proposed a single equation representation of the carbon cycle, estimated from historical data. Because of its simplicity and convenience, variants of this model are widely used in economic analyses of global climate change (for examples, see [6,13]). Unfortunately, the Nordhaus-type approach represents the behavior of the climate system only poorly. It violates first-order physical and chemical principles and it might lead to errors in predicting the carbon budget. Fortunately, however, the fundamental problems associated with the Nordhaus approach can be avoided if the information contained in complex climate models is extracted in the form of their pulse response function. It allows building cost-efficient substitutes that consist of a few equations only and represent the parent model accurately.

Joos et al. [4] have developed such a substitute. Consisting of four equations only, their non-linear Pulse Response (PR) approach is easily implemented into economic models. This paper integrates their PR representation of a full climate model into the economic analysis of the greenhouse effect. This innovation allows for the first time (1) to test the Nordhaus model against climatologists' carbon cycle models, (2) gives the economics of global climate change a better natural science foundation, and (3) reduces the uncertainties typically included in the specification of the Nordhaus climate model.

Section 2 describes in some detail the Nordhaus model and a pulse response substitute of the Bern model. Section 3 sets up a simple economic framework within which these two climate sub-models are contrasted. Section 4 carries out some computational experiments to compare optimal climate policies for different climate sub-models. Conclusions are found in section 5.

2. Climate modeling

The carbon cycle community has developed models that are based on first-order physical and chemical principles. These models are validated and checked against observations. Unfortunately, the application of these models is re-

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stricted by time and computing capacities. Even calculating atmospheric CO_2 concentrations from prescribed emissions requires a substantial effort, and a full analysis of the interaction between the climate system and the world economy is almost prohibitively costly.

2.1. The Nordhaus approach

In his pioneering work, Nordhaus [7] developed a substitute for a full carbon cycle model. It consists of a single equation and describes the relationship between carbon emissions and atmospheric CO_2 concentration in the most simple way: At any point of time, the actual stock $Q(t)$ of atmospheric carbon depends on the former one, $Q(t-1)$, as well as past period emissions, $e(t-1)$:

$$Q(t) = (1-\lambda)Q(t-1) + \varphi e(t-1). \qquad (2.1)$$

Instantaneous disappearance of carbon to an unspecified sink is represented by the parameter φ. Long-run disappearance is represented by a first-order decay rate λ. Given an assumed turnover time for deep oceans of 120 years, i.e., $\lambda = 1/120$, Nordhaus reports a best estimate for φ of 0.64. These are middle values calculated through ordinary least squares on data of atmospheric CO_2 and fossil emissions from 1850 to 1986.

In the literature (see [9]; for a more elaborate position, see [14]) there are two principal objections against the Nordhaus formula. Economists typically argue that the parameter φ is estimated inaccurately, but they do not criticize the carbon accumulation itself. On the contrary, formula (2.1) looks quite familiar to them. Global climate is viewed as environmental capital, and (2.1) is a standard accumulation equation. From a climate scientist's point of view, however, (2.1) attracts severe critics, since it lacks almost any physical basis. For example, it would be possible to destroy atmospheric carbon dioxide completely and the age structure of the atmospheric carbon stock is not reflected.

2.2. The non-linear pulse response approach

There exist alternatives that avoid the problems associated with a Nordhaus formulation. Once a full carbon cycle model has been developed, its information can be extracted by a pulse response function (Green's function) for building a simple, cost-efficient, but still accurate substitute model.

Linear pulse representations of the carbon cycle have been applied in earlier integrated assessment studies of global warming (e.g., [1–3]). However, these studies neglect the well understood non-linear relationship between partial pressure and dissolved inorganic carbon in seawater.

Non-linear pulse response functions provide a transparent and simple characterization of the climate system. The non-linear PR models are accurate with respect to their parent models. For example, the agreement between a 3-dimensional ocean general circulation model with a high-resolution terrestrial biosphere model and their pulse substitute is better than four percent for the cumulative uptake of anthropogenic carbon applying the stabilization scenarios of the Intergovernmental Panel on Climate Change (IPCC).

While it appears likely that elevated CO_2 concentrations stimulate plant growth, it is to a large degree unresolved how the interaction between elevated CO_2 levels, global warming, and additional nitrogen input will affect future terrestrial carbon storage (see [11]). This view is still valid. Furthermore, it is controversial whether the feedback between global warming and the marine carbon cycle should be included in simulations of future atmospheric CO_2. Maier-Reimer et al. [5] concluded that ocean-only models are sufficient to simulate future atmospheric CO_2.

To focus the analysis on the effects of differences in carbon cycle modeling, we suppose in the following that future carbon emissions by land use changes are balanced by a terrestrial sink and the net uptake of the land biota can be taken as negligible. That is, the development of the atmospheric CO_2 concentration can be viewed as a function of fossil carbon emissions, $e(t)$, and ocean uptake, $u(t)$, only,[1] hence the atmospheric budget equation is given by

$$Q(t+1) = Q(t) + e(t) - u(t). \qquad (2.2)$$

Again, $Q(t)$ is the stock of atmospheric CO_2 relative to the pre-industrial equilibrium.

Parameterized box-type models as well as 3-dimensional general circulation ocean models are used to calculate the oceanic uptake $u(t)$ of anthropogenic carbon. The key processes are the air–sea exchange, the dissolution and chemical reaction of CO_2 to form bicarbonate and carbonate ions, as well as the transport to depth. While carbon chemistry and the related air–sea transfer are highly non-linear, ocean transport is usually modeled by a set of linear equations and can be captured by a pulse function. Therefore the development of the surface concentration in inorganic carbon can be approximated by the convolution integral of the air–sea flux with a so-called mixed-layer pulse response function.

Normalized mixed-layer response functions, r_s, are obtained by monitoring the decrease of a perturbation in surface concentration due to an initial carbon input at time 0 using a complex ocean transport model. The value of the pulse response function r_s at any particular time is the fraction of the initially added carbon which is still found in the surface layer; the rest has been removed by ocean transport. The mixed-layer response decreases rapidly during the first few year as surface waters are rapidly exchanging with the underlying layers. Then r_s is changing slowly as deep ocean overturning is taking place on a century timescale.

r_s contains the information about the transport scheme of the ocean model in a comprehensive way. The change in surface water concentration of inorganic carbon, $\Delta Q_s(t)$, can then be represented as the sum of earlier air–sea in-

[1] Analytical representation of the response functions and the carbon chemistry as well as parameters required to build the substitute model are given in appendix II.

put fluxes, $u(t')$, at times t' multiplied by the fraction still remaining in the surface layer after time $t - t'$:

$$\Delta Q_{\mathrm{s}}(t) = \varepsilon \sum_{t'} u(t') r_{\mathrm{s}}(t - t'). \tag{2.3}$$

The conversion factor ε is used to relate ocean uptake, u, in units of ppmv to changes in the inorganic carbon concentration of the surface layer, ΔQ_{s} in μmol/kg. ε depends on the ocean area and the surface layer thickness of the model.

The net air-to-sea gas flux is a function of a constant gas transfer rate, k_{g}, and the difference in partial pressure between air and seawater. Then ocean uptake u for period t of length Δt is

$$u(t) = k_{\mathrm{g}}\big[Q(t) - \Delta p_{\mathrm{s}}(t)\big]\Delta t, \tag{2.4}$$

where Δp_{s} represents the perturbation in the surface water partial pressure from its pre-industrial equilibrium.

The perturbation in surface water partial pressure, $\Delta p_{\mathrm{s}}(t)$, and inorganic carbon content $\Delta Q_{\mathrm{s}}(t)$ is related by the chemical equilibrium between dissolved CO_2 and the bicarbonate, HCO_3^-, and carbonate, CO_3^{2-}, ions, and the CO_2 solubility, where Δp_{s} is in units of ppmv and ΔQ_{s} in μmol kg^{-1}. The non-linear relationship between $\Delta Q_{\mathrm{s}}(t)$ and Δp_{s} is calculated through a chemical equilibrium model that considers surface concentrations of inorganic carbon, alkalinity, borate, phosphate and silicate (see appendix II).

2.3. Comparison between a Nordhaus and the PR carbon cycle model

In short, the major difference between a Nordhaus and a pulse response formulation of the carbon cycle is that the Nordhaus representation neglects first-order physical principles and conflicts with models used in the carbon cycle community. In particular, it does not take the age structure of the atmospheric carbon stock fully into account, but represents the removal rate of anthropogenic carbon from the atmosphere by two time scales only. This is erroneous, since anthropogenic CO_2 is, unlike other anthropogenic greenhouse gases, not oxidized or destroyed in the climate system but redistributed between different reservoirs with a wide spectrum of overturning time scales.

To understand the difference, suppose that the climate system is shocked by a carbon input that corresponds to an instantaneous doubling of the atmospheric CO_2 concentration. As figure 1 shows, in the Nordhaus model 36% of the carbon input is removed instantaneously. The other 64% decays with a half life of 84 years, and within a time horizon of about 500 years the perturbation in atmospheric CO_2 is completely removed from the atmosphere. This means that CO_2 can either be destroyed completely or the ocean has an infinitely large uptake capacity.

In the PR model, the perturbation is decreasing fast in the first few years as a consequence of the relatively fast air–sea exchange and fast mixing in the surface ocean (again, see figure 1). Then, the removal rate diminishes as transport to the deep ocean is a slow process. The size of the ocean and the atmosphere as well as the carbonate chemistry determine the ultimate oceanic uptake capacity for anthropogenic carbon in the model. About 20% of the original pulse remains airborne; the other 80% has been taken up by the ocean. This means that anthropogenic CO_2 is partitioned between the atmosphere and the ocean according to the well-known aquatic carbonate chemistry.

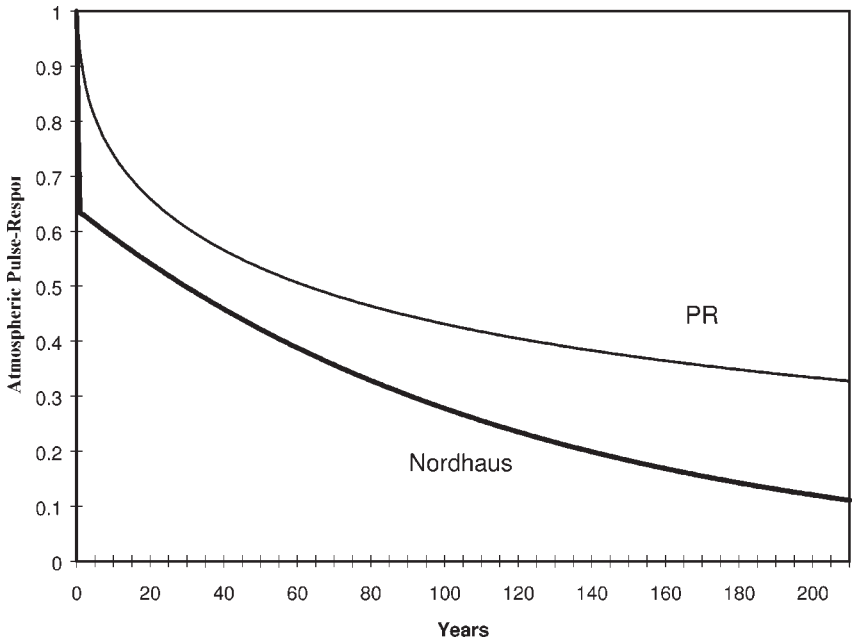

Figure 1. Reaction to instantaneous doubling of atmospheric CO_2 concentration for the PR model and the Nordhaus carbon cycle representation.

Ultimately, the equilibration between deep water and sediments would remove another 10% of the original pulse input from the atmosphere. However, this interaction is taking place on time scales of millennia which certainly is beyond the time scales considered in almost any impact study.

3. Economic modeling

Because of the important role fossil energy plays today, it is essential to analyze the interaction between the economy and the climate in a logically consistent framework. For this purpose, both the Nordhaus and the PR representation of the carbon cycle are combined with the same, stylized-facts model of the world economy.

As in [12], time is taken as discrete and each time period is one decade in length. Among the various greenhouse gases, carbon dioxide is considered as the most relevant one. Potential global warming is caused by increased atmospheric CO_2 concentration and directly affects production, but not utilities.

Economic effects of climate change are represented by a quadratic damage function

$$\Phi(t) = \big[Q(t)/\Omega\big]^2. \tag{3.1}$$

$Q(t)$ denotes the atmospheric CO_2 concentration relative to the pre-industrial carbon stock of 280 ppm. Ω marks the critical value at which production is reduced to zero.

$\Phi(t)$ is the so-called environmental loss factor. It represents climate change induced damages and measures the economic costs of global warming in terms of forgone gross product. That is, if the atmospheric CO_2 concentration is raised to levels $Q(t)$, then the productivity of factors is negatively affected in a way that, given the same inputs, only $1 - \Phi(t)$ percent of the world's potential gross production without climate change is still available.

For convenience, let the world's conventionally measured gross output $y(t)$ without climate effects be a Cobb–Douglas function of labor $L(t)$ and capital $K(t)$ inputs, respectively,

$$y(t) = \beta L(t)^{\alpha} K(t)^{1-\alpha}, \tag{3.2}$$

where β is a scaling parameter and α is the value share of labor.

To sidestep a detailed energy sub-model, CO_2 emissions are viewed as a linear function of total output. It is supposed that without greenhouse gas abatement, σ units CO_2 are emitted if one unit of conventional gross output is produced. Emissions can be reduced, however, by employing abatement activities. Therefore, instantaneous CO_2 emissions $e(t)$ are given by

$$e(t) = \big(1 - a(t)\big)\sigma y(t), \tag{3.3}$$

where $a(t)$ denotes the fraction of gross emissions abated in period t. Note that $0 \leqslant a(t) \leqslant 1$ is defined as the ratio of abated emissions to gross emissions.

As in [8], abatement costs are expressed in units of gross world output and are supposed to be quadratic in abatement activities $a(t)$:

$$m(t) = \tau\big[a(t)\big]^2. \tag{3.4}$$

The scaling factor τ is chosen such that complete elimination of CO_2 emissions consumes twenty percent of the world's gross output. This is quite a pessimistic estimate. Suppose, for example, carbon-free backstop technologies were available at marginal costs of 200 US$, then the scaling factor τ would be only 5%. Our estimate is equivalent to average costs of 800 US$ or marginal costs of 1400 US$ for carbon-free energy.

Climate change reduces economic productivity and only a fraction $1 - \Phi(t)$ of the conventional gross output $y(t)$ is at the society's disposal. "Green output" $(1 - \Phi(t))y(t)$ can be consumed, $c(t)$, invested, $b(t)$, or used for CO_2 abatement $m(t)$:

$$\big(1 - \Phi(t)\big)y(t)\big(1 - m(t)\big) \geqslant c(t) + b(t). \tag{3.5}$$

Within this model there are two capital stocks the society can control. The environmental capital stock is determined by the atmospheric accumulation of CO_2, hence depends upon the society's abatement activities $a(t)$. The development of the society's physical capital stock $K(t+1)$,

$$K(t+1) = \nu K(t) + b(t), \tag{3.6}$$

is driven by investment activities $b(t)$ and the capital depreciation rate ν. Therefore, the most simple way to look at the economics of global climate change is to suppose that optimal economic development and climate policy are governed through an intertemporal optimization process. That is, if δ is the social discount rate, then consumption, production, investment into physical capital and greenhouse gas abatement are determined as if a policy maker has maximized the discounted sum of the logarithm of consumption $c(t)$:

$$W = \sum_t \delta^{-t} \ln\big(c(t)\big), \tag{3.7}$$

subject to the above mentioned production and climate constraints.

4. Computational experiments

What are the differences in optimal greenhouse gas abatement and global climate change if either the Nordhaus or the PR formulation of a climate sub-model is used? Simulations to answer this question are computed with GAMS/MINOS and GAMS/PATH. Parameters for the computational experiments are benchmarked against 1990 data (for details, see appendix I).[2] Climate effects are calculated on an annual basis starting at 1990, whereas projections of economic variables are based on ten-year time

[2] As such 1990 is taken as starting point of the simulations except for the PR climate sub-model. PR requires a pre-run, beginning at 1900, to impose the "age-structure" on atmospheric carbon stock (see section 2).

intervals. To reduce end-of-time-horizon effects, results are reported till the end of the 21st century, but computations are carried out till 2200.

4.1. Choice of the discount rate

Abatement costs are borne early but benefits will not accrue until the distant future. This immediately leads to the issue of choosing an appropriate discount rate. The lower the rate of discounting, the higher is the present value of benefits from greenhouse gas abatement. In other words, the lower the discount rate, the higher is the weight placed upon the welfare of future generations. Therefore, the problem of choosing a discount rate is closely related to the issue of intergenerational equity and intergenerational altruism (for a detailed analysis, see [12]).

Computational experiments are carried out for two different choices of utility discount rates. A discount rate of zero marks the case of a highly altruistic policy maker. Alternatively a close-to-the-market scenario is considered and a utility discount rate of three percent is applied. This seems like only a minor difference, but it leads to a very different view of the world. With a 3% utility discount rate, rates of return on capital are similar to those that actually prevail in international capital markets. This is said to be a "descriptive" use of the model. By contrast, a utility discount rate close to 0% is illustrative of a "prescriptive" scenario. It implies that the world ought to move toward much lower rates of return capital through a rapid step-up in the near-term rate of physical investment and through rapid near-term abatement of carbon emissions (see [12]).

4.2. Results

Figures 2, 3 and table 1 show the main results of our computational experiments.

Optimal CO_2 emissions significantly depend on the choice of the climate model as well as the choice of the discount rate (see figure 2). If future welfare is discounted at 3%, then the Nordhaus model would allow emissions to triple by the end of the next century. Given the same discount rate, the PR approach projects a turn around in emissions already at the middle of the next century. This suggests that the Nordhaus model systematically underestimates the accumulation of atmospheric carbon.

Almost identical atmospheric carbon stocks are projected for both sub-models, if the discount rate is low (see figure 3). The explanation is quite obvious. If the discount rate is low, then there is a huge investment both in environmental and physical capital at the early time horizon. Independent of the carbon cycle representation, optimal abatement policy does only allow for a relatively modest growth in atmospheric CO_2 and the projected concentrations can not be that different.

With discounting at a market-rate abatement activities are lower and optimal emissions are significantly higher. Since each unit of carbon emitted leads to higher atmospheric CO_2 in the PR than in the Nordhaus model, climate policy that balances present values of marginal costs

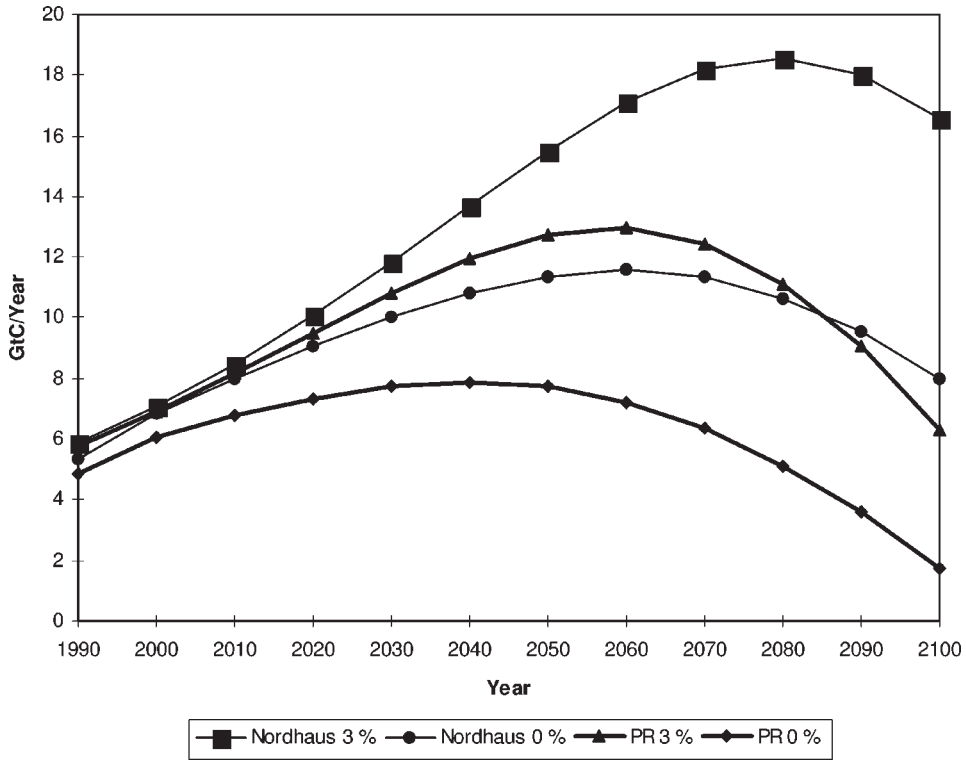

Figure 2. Optimal CO_2 emissions for utility discount rates of 0 and 3% with a pulse response (PR) or the Nordhaus carbon cycle representation.

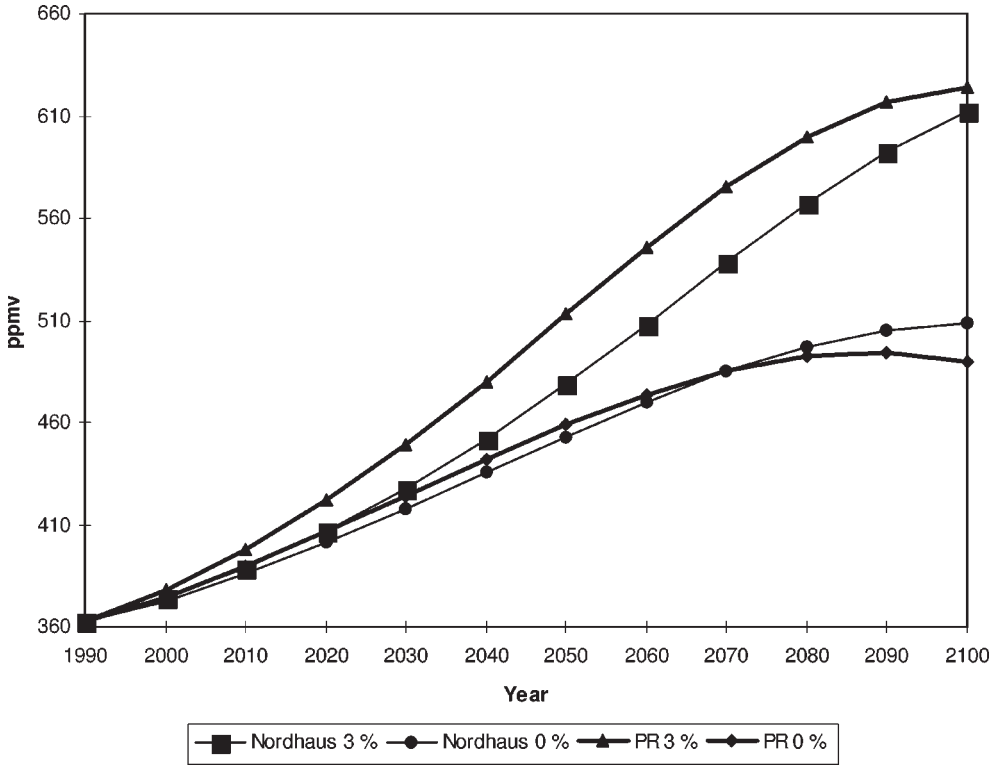

Figure 3. Atmospheric carbon concentration calculated by the PR and Nordhaus approaches.

Table 1
Consumption and abatement levels.

Year	Consumption			Abatement level		
	PR	Nordhaus	Δ (%)	PR	Nordhaus	Δ (%)
3% discounting						
1990	18.7	18.6	0.43	0.03	0.02	48.40
2030	40.5	40.4	0.27	0.16	0.09	45.61
2070	76.3	77.2	−1.17	0.54	0.34	36.94
2100	114.3	126.6	−10.73	0.87	0.66	24.32
0% discounting						
1990	15.5	15.3	1.09	0.18	0.10	43.40
2030	40.4	41.9	−3.63	0.47	0.31	32.35
2070	77.2	82.7	−7.07	0.80	0.64	19.65
2100	126.6	133.7	−5.6	0.97	0.86	11.52

Note: Δ (%) is the difference between PR and Nordhaus projections relative to the PR projection.

of abatement against benefits must imply relatively lower emissions (higher abatement costs) and higher benefits from avoiding damages with the pulse response carbon cycle representation as compared to the Nordhaus approach.

For the two different values of the discount rate, table 1 reports consumption and abatement activities depending on the choice of the respective climate model. In any case, a Nordhaus model suggests higher consumption levels in the long run. But the differences occur very late. At a discount rate close to market conditions it is not until 2070 that projected consumption differs significantly. With a high degree of altruism, differences become obvious a bit earlier, but remain quite small even at the end of the next century. However, a significant difference in greenhouse gas abatement is observed. With the PR model, already in 1990 about three percent of emissions should be abated.

In summary our simulations reveal that in a Nordhaus framework optimal emissions are always higher than in the PR model. Consequently, abatement always starts later and consumption is higher if the Nordhaus climate model is employed. It is interesting to note, however, that differences in economic effects expressed in per capita consumption are almost negligible.

5. Conclusions

Our CGE simulations support Schimel's [10] conjecture that errors in modeling the carbon budget can come at high

costs. Contrasting the Nordhaus representation of the carbon cycle with a non-linear PR approach has revealed significant differences in optimal carbon emissions. With a non-linear PR model optimal CO_2 emissions are always below those that are regarded to be optimal in a Nordhaus climate sub-model. Not surprising then, the PR approach suggests higher abatement levels to be optimal, and the economists' policy recommendations in terms of optimal carbon emission might change significantly when the Nordhaus model is replaced by a more elaborate climate sub-model.

Future research on this topic is strongly indicated, since most of the economic policy recommendations are based on the Nordhaus model. In particular, the over-simplified treatment of economic damages should be improved. Costs are related to the changes in climate variables such as temperature, precipitation, sea level rise, and ice sheet melting rather than carbon stocks and a sub-model of the biosphere would allow for more precision in the carbon budget. And finally, one has to note that climate policy is not a one-for-all-times decision. Allowing for a correction of carbon targets when prediction errors occur can shed further light on optimal greenhouse policy, in particular in environments that are characterized by uncertainty about the validity of climate models.

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Appendix I: Benchmark data and model parameters for the economy model

Data	
y, GDP 1990 (US$ trillions)	22.9
b, investments 1990 (US$ trillions)	5.7
K, capital stock 1990 (US$ trillions)	63
L, labor stock 1990	1
c, consumption 1990 (US$ trillions)	17.2
Annual potential growth rate of GDP (%)	2
ν, annual capital depreciation rate (%)	5
α, labor value share of GDP 1990 (%)	72.5
β, scaling factor for GDP function	1.809
τ, scaling factor for abatement cost function	0.2
σ, scaling factor for carbon emission function (GtC per US$ trillion)	0.26
Ω, critical perturbation in atmospheric CO_2 (ppmv) relative to 1990 level	801

Note: Labor is measured in efficiency units. Annual potential growth of GDP reflects growth in efficiency units.

Appendix II: Model parameters and analytical functions for the pulse response model

This study uses the pulse substitute representation of the HILDA ocean model. The HILDA model is the oceanic component of the Bern model that has been used as a standard for IPCC calculations (e.g., [11]). The HILDA model is a spatially aggregated, parameterized model. Air–sea and ocean transport are described by six parameters which are determined such that both the oceanic distribution of natural and bomb-produced radiocarbon are reproduced at the time of the Geochemical Ocean Section Study survey (see [11], Version $K(z)$).

The model was tested by comparing simulated versus observed distributions of CFCs and argon-39. The parameterization of surface-to-deep transport by eddy diffusion was evaluated using the Princeton/GFDL general circulation ocean transport model. Simulated average ocean uptake is 2.1 GtC/year for the 1980–1989 period and thus close to the IPCC estimate of 2.0 ± 0.8 GtC/yr [11].

k_g, air–sea gas transfer rate (1/year)	$1/9.06$
ε, conversion factor (μmol (kg ppm)$^{-1}$)	6.343

Normalized pulse response function of the HILDA ocean model

For $0 < t \leqslant 2$ yr:

$$\begin{aligned} r_s(t) = {} & 0.12935 + 0.21898 \exp(-t/0.034569) \\ & + 0.17003 \exp(-t/0.26936) \\ & + 0.24071 \exp(-t/0.96083) \\ & + 0.24093 \exp(-t/4.9792) \end{aligned}$$

(check value: $r_s(t = 2\text{ yr}) = 0.32071$).

For $t > 2$ yr:

$$\begin{aligned} r_s(t) = {} & 0.022936 + 0.24278 \exp(-t/1.2679) \\ & + 0.13963 \exp(-t/5.2528) \\ & + 0.089318 \exp(-t/18.601) \\ & + 0.037820 \exp(-t/68.736) \\ & + 0.035549 \exp(-t/232.30) \end{aligned}$$

(check value: $r_s(t = 2\text{ yr}) = 0.32068$).

Carbonate chemistry function

For $0 \leqslant \Delta p_s \leqslant 1320$ ppm and $T = 18.2\,^{\circ}$C:

$$\begin{aligned} \Delta p_s = {} & (1.5568 - 1.3993 \times 10^{-2} \times T)\Delta Q_s \\ & + (7.4706 - 0.20207T) \times 10^{-3} \times (\Delta Q_s)^2 \\ & - (1.2748 - 0.12015T) \times 10^{-5} \times (\Delta Q_s)^3 \\ & + (2.4491 - 0.12639T) \times 10^{-7} \times (\Delta Q_s)^4 \\ & - (1.5468 - 0.15326T) \times 10^{-10} \times (\Delta Q_s)^5; \end{aligned}$$

Δp_s is in units of ppmv, ΔQ_s in μmol kg^{-1}.

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