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A note on Quaternary climate modelling using Boolean delay equations

DG Wright[1], TF Stocker[2], and LA Mysak[2]

[1] Department of Fisheries and Oceans, Bedford Institute of Oceanography, Dartmouth, Nova Scotia B2Y 4A2, Canada
[2] Centre for Climate and Global Change Research, and Department of Meteorology, McGill Unversity, 805 Sherbrooke Street West, Montreal, Quebec H3A 2K6, Canada

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Abstract. The Boolean delay equation model of Ghil et al. for the study of Quaternary ice ages has been reexamined and also extended to include a hydrological feedback mechanism that directly influences the thermohaline circulation. For the basic Ghil et al. model (with the original and with corrected time delays), we show that the maximum duration for a high ice volume state depends continuously but not monotonically on the time delays of the problem. In particular, both short spiky glacials and longer glacials can be obtained from identical parameter values by simply choosing different initial conditions. In the extended model, we find that an additional temperature-hydrology-ocean feedback mechanism tends to generate longer glacials, but in neither case do the average time scales of model variability compare favourably with those of the major Quarternary glacials.

Introduction

Over the past 20 years a large number of climate models of various complexity have been developed. Although the most advanced models represent the present climatic state reasonably well, still unsolved conceptual questions on the behaviour of the climate system require the development and application of simpler climate models. Ghil et al. (1987), henceforth referred to as GMP, have recently proposed an approach to study Quaternary climate variability that uses a highly idealized climate model. In GMP the climate system is described by three state variables, namely, a global air temperature T, a Northern Hemisphere ice volume V, and the intensity C of the thermohaline circulation. These variables take only the Boolean values 0 or 1, correspondingly to a low or high value of a state variable. Thus in this framework, a glacial period would be represented by $V=1$ and an interglacial by $V=0$. In GMP conceptual relations among these three state variables are expressed in terms of Boolean delay equations (Ghil and Mulhaupt 1985).

In spite of the above simplification of the processes associated with the late Quaternary ice ages, the GMP approach has appeal for two reasons. First, the framework of Boolean delay equations (BDEs) gives a rigorous method of modelling a network of feedback mechanisms which characterize interactions in the climate system (Kellogg 1983). Second, the requirement for detailed information on the physical processes considered is significantly reduced in the BDE formulation.

In the classical modelling approach based on differential equations, interaction parameters are often difficult to determine from the basic physical laws and yet they have a crucial influence on the solution. This difficulty does not arise with the highly idealized approach of BDEs since only a knowledge of the characteristic time scales and the relevant feedback mechanisms is required. On the other hand, BDE models are severely limited since they do not deal with a smooth time evolution of the climate system and they represent a present climatic state as being influenced by the state of the system at particular, isolated times in the past. The fact that the model variables take Boolean values seems to preclude the consideration of any processes with a cumulative effect over time intervals in the past.

It should be emphasized that we believe BDEs may be useful to provide a first and *preliminary* answer to conceptual questions concerning climate variability and climate processes, but by no means do they replace or substitute for any quantitative approach based on differential equations. As will be demonstrated, even in the simplified BDE framework, results are not unambiguous and their qualitative nature can strongly depend on the initial conditions.

The purpose of this note is to correct and extend the results presented in GMP concerning deep water formation and Quaternary glaciations. The major conclusion reached by GMP, that the oceanic deep water formation must have been reduced in glacial times, is reexamined. The next section describes the BDE equa-

Offprint requests to: LA Mysak

tions for two climate models: in the first model the equations are identical to those in GMP except for a correction to two of the arguments in GMP's Eq. (6); in the second model, the third of GMP's basic equations is modified to include a simple temperature-hydrology-ocean feedback mechanism. Some numerical results of the models are presented in the third section, together with an investigation of the qualitative structure of the solution time series as a function of the oceanic deep water formation time scale. The final section gives our conclusions. Also, the problems associated with the attempt at climate modelling using highly idealized BDEs are summarized.

BDE models

The GMP model

Ghil et al. (1987) begin with some simple conceptual ideas on the relationships between three state variables of the global climate system: the global air temperature, T; the volume of the Northern Hemisphere ice sheets, V; and the intensity C of the thermohaline circulation that carries cold bottom waters to the surface of the world ocean. These relations are expressed in terms of Boolean delay equations wherein each variable is assumed to be in one of two possible states. These are referred to as 'high' (represented by a 1) and 'low' (represented by a 0). The equations considerd by GMP are:

$$V(t) = T(t - \tau_{acc}) \tag{1}$$

$$T(t) = \bar{V}(t - \tau_{if}) \wedge \bar{C}(t - \tau_{dw}) \tag{2}$$

$$\bar{C}(t) = V(t - \tau_{dw}) \wedge \bar{V}(t) \tag{3}$$

where t is the time measured in ka (1 ka = 1000 years), and ($^-$) and $\wedge$ stand for the Boolean operators *not* and *and,* respectively.

This set of equations represents logical relations between the three climate variables and describes three operating feedback mechanisms. Equation (1) states that a change to the high (low) ice volume state follows a change to the high (low) global air temperature state after a time interval τ_{acc}, required for an appropriate volume of ice to accumulate (deteriorate). The essence of Eq. (1) is that increased temperatures lead to a more active hydrological cycle which, among other effects, leads to an increase in the accumulation of ice. References supporting this relationship are given in GMP.

Equation (2) expresses the temperature-albedo feedback, which takes into account the thermohaline circulation. If the ice volume is low (i.e., the albedo is reduced) and the movement of cold bottom water to the ocean surface is inhibited, then the global mean temperature will be high. Otherwise, the temperature will be in the low state. τ_{if} is the time required for a large volume of ice to expand through viscoplastic flow to cover an area so as to increase the earth's surface albedo sufficiently to shift temperatures to the low state. τ_{dw} is the time required for a change in C to influence T. GMP refer to τ_{dw} as a convective overturning time for the oceans.

Equation (3) is logically equivalent to Eq. (5) from GMP. It can be interpreted as follows: if a large volume of ice melted over the last τ_{dw} time interval (thereby producing a low ice state), then the resulting fresh water runoff will stabilize the high latitude water column and reduce convective overturning which shifts C into its low state. For the purpose of cross-referencing, we note that the above Eqs. (1) and (2) corresponding to (2) and (4) in GMP.

Substituting Eq. (3) into the RHS of (2) and then the resulting equation into the RHS of Eq. (1), we obtain a BDE for V alone:

$$V(t) = \bar{V}(t - \theta_1) \wedge V(t - \theta_2) \wedge \bar{V}(t - \theta_3) \tag{4}$$

where $\theta_1 = \tau_{acc} + \tau_{if}$, $\theta_2 = \tau_{acc} + 2\tau_{dw}$ and $\theta_3 = \tau_{acc} + \tau_{dw}$. Equation (4) is equivalent to GMP's Eq. (6) except that here θ_2 and θ_3 contain the extra term τ_{acc} which was inadvertently omitted by GMP. The corrected model does not exhibit the separation of surface time scales, represented by τ_{acc} and τ_{if}, and the ocean time scale τ_{dw}. Also, θ_2 and θ_3 take on larger values. Henceforth it is these corrected θ_2 and θ_3 values that we shall refer to, unless otherwise specified. For all numerical simulations of (4) presented here, $V(t)$ will be periodic with period θ_2 after the transients die away (Ghil and Mulhaupt 1985). Usually, the transient behaviour disappears within one period, but occasionally, very long transient intervals lasting for several periods can occur.

It is useful to examine the maximum duration of glaciation or high ice volume V which is implied by (4). This is given by

$$J_G = \min(|2\tau_{dw} - \tau_{if}|, \tau_{dw}). \tag{5}$$

To establish this result, let $t = t_0$ be a time after the transients have died out with $V(t_0) = 1$. Then according to

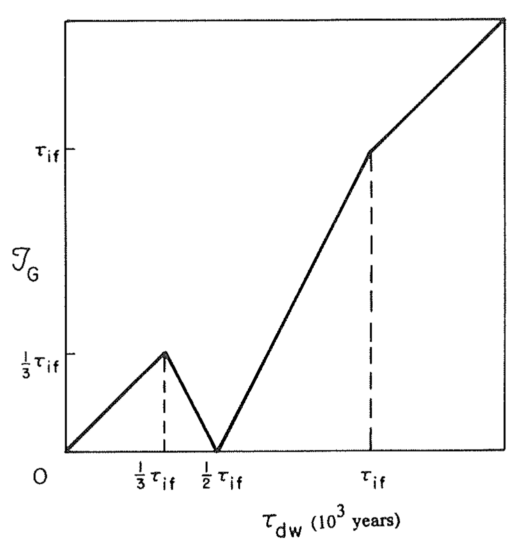

Fig. 1. Maximum duration of glacial, $\mathcal{J}_G$, for the corrected GMP model (4)

(4) the ice must be heavy at $t=t_0-\theta_2$ and light at $t=t_0-\theta_1$ and $t_0-\theta_3$. This must be true for any t_0, and hence for any $t_0-\theta_2$ within a heavy ice interval. Thus the heavy ice interval cannot exceed the minimum of $|\theta_2-\theta_1|=|2\tau_{dw}-\tau_{if}|$ and $\theta_2-\theta_3=\tau_{dw}$, which yields the desired result.

Figure 1 shows the behaviour of the maximum glacial duration J_G as a function of τ_{dw} given by (5) for $\tau_{dw}\geq 0$. We note that outside the interval $\tau_{if}/3<\tau_{dw}<\tau_{if}$, $J_G=\tau_{dw}$. However, inside this interval, $J_G<\tau_{dw}$, with $J_G=0$ (no glaciation) when $\tau_{dw}=\tau_{if}/2$, which corresponds to $\theta_1=\theta_2$. GMP estimate that typically $\tau_{if}=3$ ka; hence if one restricts attention to integer values of τ_{dw} in the above interval, it is only for $\tau_{dw}=2$ ka that exceptionally short glacial periods are obtained. Equation (5) gives only an upper bound. Short glacials of 0(1 ka) can occur for both $\theta_2<\theta_1$ and $\theta_2>\theta_1$, which will be demonstrated in the results section.

An extension of the GMP model

GMP state that a positive feedback mechanism operates between ice volume and temperature as a result of two competing effects. An increased temperature favours ablation of ice sheets, but it is argued that this is outweighed by a more active hydrological cycle during the warmer climate. This yields increased precipitation and snow accumulation on the ice sheets. The hydrological cycle has a significant effect also on deep water formation through river runoff and evaporation minus precipitation. This suggests linking the hydrological cycle directly to deep water formation and hence the thermohaline circulation. Consider a new climatic variable $H(t)$ that represents the strength of the hydrological cycle. $H=1$ would describe a climatic state in which the poleward atmospheric water transport due to evaporation minus precipitation and river runoff are increased. Equation (3) for the ocean circulation intensity C is thus expanded into the form

$$\bar{C}(t)=\{V(t-\tau_{dw})\wedge\bar{V}(t)\}\vee H(t-\tau_{hy}), \quad (6)$$

where $\vee$ denotes the Boolean *or* operator. Equation (6) states that the thermohaline circulation is reduced by an input of fresh water stabilizing the water column. This fresh water is either due to ice sheet melting or to an enhanced hydrological cycle. The adjustment time scale of the latter is τ_{hy}. As noted by GMP, the hydrological cycle is stronger when the global air temperature increases and thus

$$H(t)=T(t-\tau_{hy}) \quad (7)$$

is proposed as a logic connection between H and T. The latter choice is also consistent with an enhanced temperature increase which will also act to reduce the thermohaline circulation during periods of high T. This is the temperature-hydrology-ocean feedback mechanism.

Putting (7) into (6), using (1) and (2) to eliminate $\bar{C}$ and T, the model equation for V is given by

$$V(t)=\{\bar{V}(t-\theta_1)\wedge V(t-\theta_2)\wedge\bar{V}(t-\theta_3)\}\vee\{\bar{V}(t-\theta_1)\wedge V(t-\theta_4)\}, \quad (8)$$

where $\theta_4=\tau_{dw}+2\tau_{hy}$. Assuming that the hydrological time scale τ_{hy} is at least and order of magnitude shorter than τ_{dw}, we approximate θ_4 by τ_{dw}. In the next section Eqs. (4) and (8) are examined numerically for $\tau_{acc}=10$ ka, $\tau_{if}=3$ ka (values in GMP) and varying τ_{dw}.

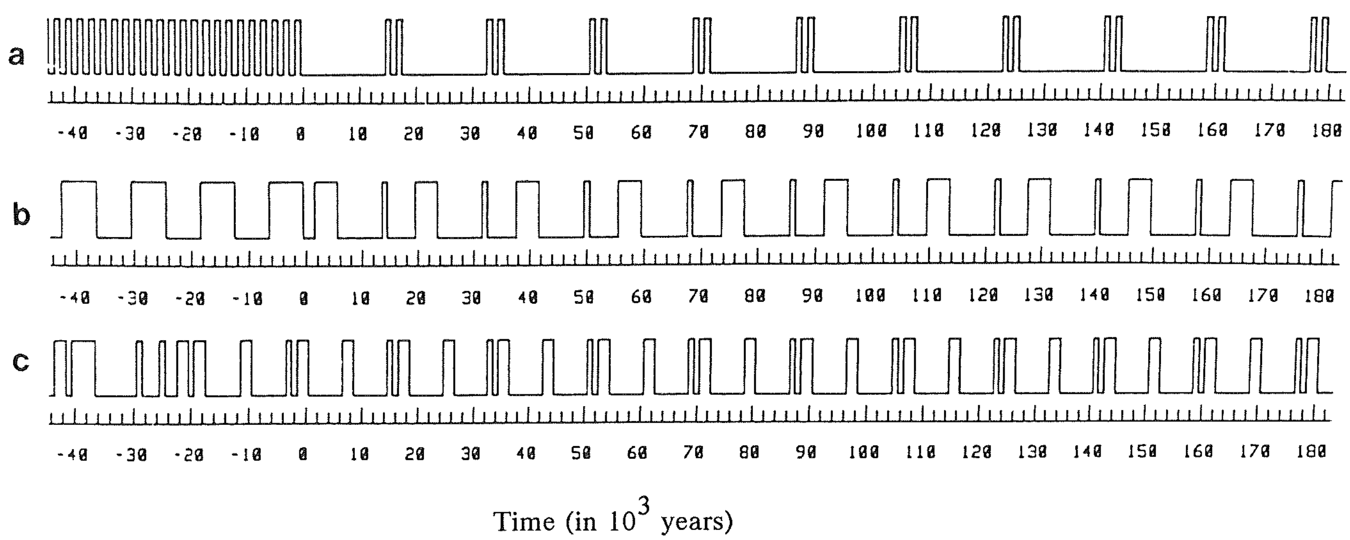

Fig. 2. Three qualitatively different time series of the ice volume V as given by Eq. (4) (the corrected GMP model) evolving from different initial conditions. These are at negative values on the time axis which is in units of 1 ka (1 ka = 1000 years). Series (*a*) and (*b*) correspond to periodic initial conditions with periods of 2 ka and 12 ka, respectively. Whereas the response in (*a*) is spiky with two neighboring short glacials of 1 ka each, the longer glacials last 4 ka in (*b*). A different response is observed when the initial conditions are random values, (*c*). The parameters used in the corrected GMP model are $\tau_{acc}=10$ ka, $\tau_{if}=3$ ka and $\tau_{dw}=4$ ka, so that $\theta_1=13$ ka, $\theta_2=18$ ka, and $\theta_3=14$ ka. Thus according to Eq. (5), the maximum possible duration of glacials is 4 ka

Results

Figure 2 shows the time series of the ice volume $V(t)$ as given by (4) for three different initial conditions. The initial conditions in Fig. 2a and b are periodic with periods of 2 and 12 ka, respectively, and the initial condition in Fig. 2c is a random Boolean sequence. It is evident that for the same parameter values τ_{acc}, τ_{if} and τ_{dw}, the resulting time series can look qualitatively different, although each has a periodicity of $\theta_2 = 18$ ka as anticipated. Figure 2a exhibits a spiky series with times of high ice volume being relatively rare and of short duration. GMP associate this qualitative behaviour with the case $\theta_2 < \theta_1$, but it appears that it also occurs for $\theta_2 > \theta_1$, the case in Fig. 2a.

Since GMP emphasize the qualitative difference in the solutions corresponding to $\theta_2 < \theta_1$, and $\theta_2 > \theta_1$, we consider the behaviour of our Eq. (4) as a function of τ_{dw}. A simple measure of whether the ice volume shows spiky events rather than extended glacials is $J_v(\tau_{dw})$, the average duration of high ice volume ($V = 1$) per period θ_2. Choosing $\tau_{acc} = 10$ ka and $\tau_{if} = 3$ ka, $\theta_1 = \theta_2$ when $\tau_{dw} = 6.5$ ka and 1.5 ka for the GMP and corrected GMP models, respectively. The averages in Fig. 3, which show J_v for these models, are calculated from 500 runs with random initial conditions. As Fig. 3 demonstrates, J_v for the GMP model is a fairly smooth function of τ_{dw} except at $\tau_{dw} = 13$ where the model equation (4) collapses to a simpler BDE because $\theta_3 = \theta_1$. The glacial duration per period θ_2 for the corrected model shows no bifurcation behaviour in either model when $\theta_2 = \theta_1$. At $\tau_{dw} = \tau_{if}$ and $\tau_{dw} = \tau_{acc} + \tau_{if} = \theta_1$ the average glacial duration is increased in the corrected GMP model. It appears that $J_v \approx 0.25$, corresponding to average glacial durations of $\tau_{dw}/2$, may be an asymptotic value for $\tau_{dw} \gg \tau_{acc}$.

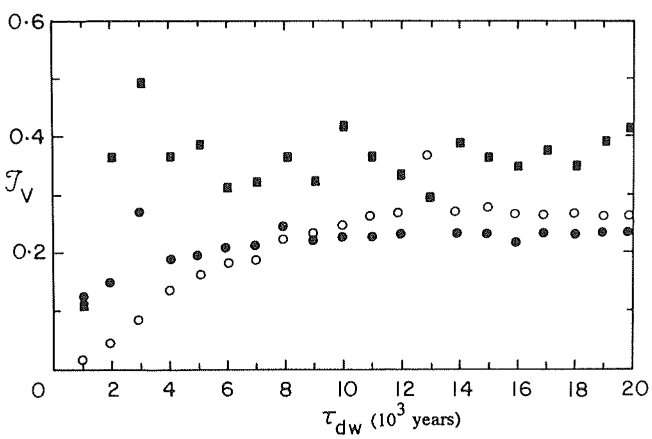

Fig. 3. Average duration $\mathcal{J}_V$ of high ice volume ($V = 1$) per period θ_2 for the three models: GMP model (*open circles*); corrected GMP model (*solid circles*); extended GMP model (*squares*). In each case the average is calculated from 500 runs with random initial conditions. The parameters used are $\tau_{acc} = 10$ ka and $\tau_{if} = 3$ ka. $\theta_1 = \theta_2$ corresponds to $\tau_{dw} = 6.5$ ka in the GMP model, and to $\tau_{dw} = 1.5$ ka in the corrected GMP model

Figure 3 also shows that significantly higher values of J_v result from the BDE model (8) which includes the temperature-hydrology-ocean feedback. For a wide range of values of τ_{dw}, the average glacial duration is of the order of 0.35.

Conclusions

The BDE Quaternary ice age model proposed by Ghil et al. (1987) has been re-examined after making a correction, and a simple extension has been presented. The main conclusion of GMP was that, in order to produce a time series of global ice volume qualitatively resembling observed proxy data, the deep ocean circulation must have been reduced during glacial times. They also argued that if $\theta_2 < \theta_1$, the system behaves qualitatively different than when $\theta_2 > \theta_1$. This conclusion is not supported for either the original GMP model or its corrected version, and it is shown that the maximum duration for high ice volume depends continuously on the time delays of the problem. Plausibly long glacials are only obtained for very long time delays τ_{dw}. Moreover, the two qualitative patterns, spiky short glacials and longer glacials, can be obtained for identical parameter values by simply choosing different initial conditions.

An extension of the BDE model has been proposed which includes a hydrological feedback mechanism influencing deep ocean circulation. A strong hydrological cycle tends to stabilize the oceanic water column because of the increased fresh water input. When combined with the other feedback mechanisms operating in the model, this leads to an increased duration of the glacials.

Although BDEs offer a rigorous framework to examine a network of climatic feedback mechanisms by characterizing each one with a particular time delay, a few words of caution follow. First, the feedback processes presented here do not account for the climatic behaviour observed during the last 500 ka. It is well documented that the last few glacials lasted for about 50 to 80 ka, which is an order of magnitude longer than the glacials modelled by the BDE models presented here. Second, much of the glacial-interglacial variability is due to orbital forcing, which is also absent in the above formulation. It is unclear how one could introduce periodic forcing into these models in a consistent way. Any forcing in this Boolean framework can only be of the same magnitude as the internal processes. Third, although the procedure for analyzing a set of BDEs is rigorous once the model feedbacks are set up, it is very difficult to decide on a reasonable set of feedback mechanisms in the first place. It is at this point that the modeller is confronted with a certain arbitrariness and thus might face an even more difficult task than that associated with developing a continuous model based on the fundamental dynamic and thermodynamic balances.

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