Pulse response functions are cost-efficient tools to model the link between carbon emissions, atmospheric CO$_2$ and global warming

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Abstract. The development of models to simulate the redistribution of anthropogenic carbon and the climate response to greenhouse gas forcing requires a substantial effort. The behavior of such models can be characterized by pulse response functions which allows one to build simple substitute models. We have used mixed-layer pulse response functions to describe the surface-to-deep mixing of tracers in the ocean and biospheric decay response functions to describe carbon turnover in the land biota. We build a simple carbon cycle-climate model using response functions. For the Princeton-GFDL ocean model, we find that the agreement between the complete model and its pulse substitute model is better than 4% for the cumulative uptake of anthropogenic carbon applying the IPCC stabilization scenarios S450 and S750. We have simulated the transient temperature response to an increasing CO_2 concentration (1% yr^{-1}) prescribing climate sensitivities of 2.1 and 4.6 K for a CO_2 doubling in the substitute model. We find a global surface temperature warming of 1.6 and 2.4 K after 80 years in good agreement with the corresponding results (1.5 to 2.7 K) of 9 atmosphere-ocean general circulation models (Houghton et al., 1996). The pulse model is efficient; a 1000 year simulation of the pulse substitute model requires 25 seconds of CPU time on a workstation.

1 Introduction

In climate science interdisciplinary co-operation is necessary to assess the global warming problem and potential impacts on the socio-economic system. However, quantitative information transfer between different disciplines is often hampered by the complexities of tools and models used by specialists. For example, the calculations of atmospheric CO_2 concentration from prescribed emission, or vice versa, requires a substantial

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effort of model-building and computer programming, even for models of simple structure. A typical example of inadequate interdisciplinary collaboration found in literature is then the coupling of a complex economic model to a very poor representation of the carbon cycle to estimate the abatement and mitigation cost of climate change. Such difficulties may be overcome if the information contained in complex models is extracted in form of their pulse response function (Green's function) (e.g., Hasselmann et al., 1996; Joos et al., 1996; Siegenthaler and Oeschger, 1978). The pulse response function can then be used to build a simple cost-efficient, but still accurate substitute model.

The theoretical justification for the use of pulse response functions is that the dynamics of a linear system is fully characterized by its pulse response function. Many complex models behave in a (approximately) linear way. Even non-linear systems may be approximated by pulse response functions as long as perturbations are relatively small. On the other hand, the application of pulse response functions is limited to the quasi linear range of a complex model.

In this paper, we will outline the general idea of pulse response function models. First, we will introduce the concept and limitations of pulse response functions by applying atmospheric response functions to calculate the redistribution of carbon in the climate system and discuss its limitations. Then, we will address the use of 'mixed-layer pulse response functions' as a new tool to substitute simple box-type models as well as complex Ocean General Circulation Models (OGCM). Next, a biospheric decay response function is introduced to describe enhancement of terrestrial carbon storage by elevated CO_2 levels. Finally, we will use pulse response functions to build a simple climate model. Technical details and a more comprehensive description of the applications of mixed-layer and biospheric decay response functions in carbon cycle modeling are given elsewhere (Joos et al., 1996). Joos et al. (1996) provide analyti-

cal expressions of the pulse response functions for four different ocean models and a biosphere model, as well as a parameterization of the carbonate chemistry which allows one to build a pulse substitute model.

2 The redistribution of anthropogenic carbon in the climate system

2.1 Atmospheric pulse response function model

Carbon cycle models are used for understanding the fate of anthropogenic CO_2, in particular to calculate carbon uptake by the ocean and the land biota. The question of primary interest is usually how much of the emitted carbon will remain airborne. The development of the atmospheric concentration can then be approximated by the convolution integral of the emission history with the atmospheric pulse response function (Siegenthaler and Oeschger, 1978). Normalized atmospheric pulse response functions, r_a, are usually obtained by monitoring the decrease of an atmospheric CO_2 perturbation due to an initial carbon input at time 0 using a complex carbon cycle model. The value of the pulse response function at any particular time is the fraction of the initially added carbon which is still found in the atmosphere. Depending on whether the pulse function was derived from an ocean-atmosphere or from a coupled ocean-atmosphere-biosphere model the removed fraction corresponds to the ocean uptake or to the carbon uptake by the ocean plus the biosphere. The atmospheric CO_2 concentration $c_a(t)$ can be represented as the sum of earlier emissions, e, at times t' multiplied by the fraction still remaining airborne after time $t - t'$.

$$c_a(t) = \int_{t_0}^{t} e(t') \cdot r_a(t - t') \cdot dt' + c_a(t_0) \qquad (1)$$

$c_a(t_0)$ is the atmospheric CO_2 concentration at a time when the system was in equilibrium. In practice, the integral is approximated by a simple sum, e.g. using a time step of 1 year.

This is a conventional and widely used approach to calculate the development of atmospheric CO_2 concentration and the global oceanic carbon uptake for a given scenario. Unfortunately, the non-linearity of the carbonate chemistry in the ocean limits the accuracy of this procedure significantly for CO_2 concentration exceeding the pre-industrial level substantially (e.g. by more than 50%). Another drawback of the atmospheric pulse response model is that atmospheric pulse response functions are in general different for different tracers. Thus, one can for example not use the same response function to calculate the oceanic uptake of carbon, CFCs or heat.
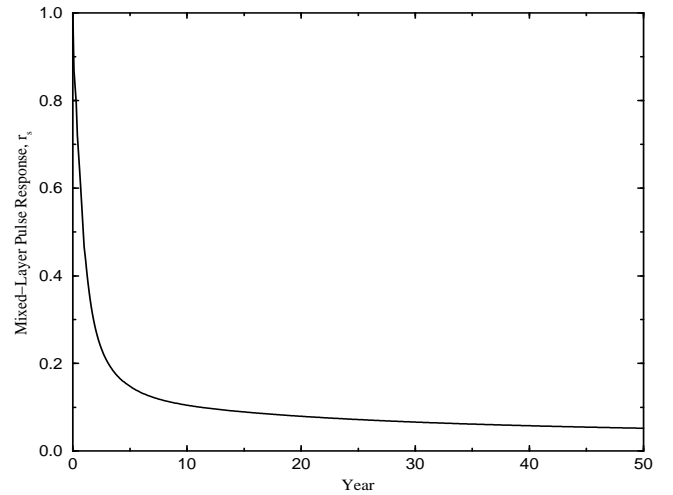

Figure 1. Mixed-layer pulse response function as determined for the Princeton-GFDL OGCM. The response was calculated by prescribing atmospheric CO_2 and the net air-sea flux as obtained from an atmospheric pulse experiment and then deconvolving Eq. 2 to 4. The value of the response function corresponds to the fraction which still remains in the surface layer after a pulse input at time t=0. 90 % of the amount added to the surface layer is removed within 10 years by mixing to deeper layers. The pulse response function approaches then on the time scales of deep ocean mixing (centuries) its asymptotic value. The latter is the ratio between surface layer depth to average ocean depth and corresponds to a well mixed ocean.

2.2 Mixed-layer pulse response function model

The problems associated with atmospheric response functions used to substitute ocean models resides in the air-sea coupling. For example, the non-linearity of the atmosphere-ocean CO_2 system lies in the transition from CO_2 to HCO_3^- and CO_3^{--} that occurs when CO_2 dissolves in seawater. On the other hand, the transport of excess CO_2 and of other passive and conservative tracers within the ocean is described as a set of linear equations and can therefore be exactly captured by pulse response functions.

We first consider the ocean uptake of carbon. The surface water concentration, c_s, is a function of the input history of carbon, i.e. the air-sea flux, f_{as} of carbon per unit area, and the surface-to-deep mixing here represented by a mixed-layer pulse response function, r_s:

$$c_s(t) = \frac{1}{h} \int_{t_0}^{t} f_{as}(t') \cdot r_s(t - t') \cdot dt' + c_s(t_0) \qquad (2)$$

where h is the depth of the surface layer. At time t_0 the surface layer is in equilibrium with the deep ocean. Again, the integral can be approximated as a sum. The mixed-layer pulse response function describes the amount of tracer which is still found in the mixed-layer after time $t - t'$ has passed since a tracer input at time t' (Fig. 1). It contains the information about the mixing scheme of the ocean model in a comprehensive form. Equation 2
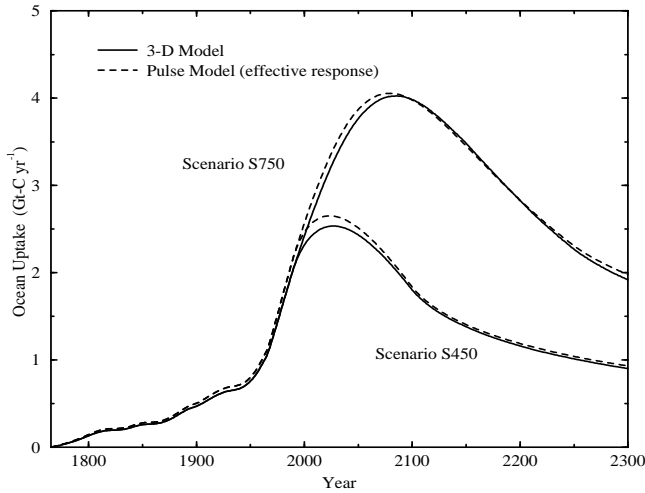
Figure 2. Oceanic carbon uptake for two different scenarios as simulated with the Princeton-GFDL OGCM (solid) and by the mixed-layer substitute model (dashed). Atmospheric CO_2 is prescribed in order to stabilize concentration at 450 and 750 ppm (Schimel et al., 1994).

is combined with equations describing air-sea exchange, an atmospheric budget equations and an equation describing the carbonate chemistry. The air-sea exchange is a function of gas transfer velocity, k_g, and the difference in the partial pressure, pCO_2, between air and seawater:

$$fas(t) = k_g \cdot [\delta pCO_{2,a} - \delta pCO_{2,s}] \qquad (3)$$

where δ represents the perturbation from the pre-industrial equilibrium. The change in atmospheric carbon inventory is given as the difference between emissions into the atmosphere-ocean system, e, and the uptake by the ocean:

$$\frac{d}{dt}\delta pCO_{2,a} = e(t) - f_{as}(t) \cdot A_{oc} \qquad (4)$$

where A_{oc} is the ocean surface area. Finally, the relationship between the change in surface water $pCO_{2,s}$ and the surface concentration of total carbon, c_s, can be described by a simple analytical expression (see Joos et al., 1996). Thus, we have a system of only 4 equations describing the ocean uptake of carbon. For other tracers, Eq. 3 can be replaced by an appropriate air-sea exchange formulation. The response function describing surface-to-deep mixing remains the same.

Figure 2 compares the ocean uptake as calculated by the described mixed-layer pulse model with that of the complete Princeton-GFDL OGCM (Sarmiento et al., 1995) for the IPCC stabilization profiles S450 and S750 (Schimel et al., 1994). It is assumed in both models that the ocean circulation and the natural marine carbon cycle do not vary with time. The agreement between the OGCM

and its pulse substitute model is better than 4% for the cumulative ocean uptake of anthropogenic carbon for the 1765-2300 period. Also the temporal evolution as calculated by the OGCM is well captured by the pulse model. Differences between the two models are much smaller than general uncertainties of oceanic carbon uptake which are estimated to be of order 40% for the last decade (Schimel et al., 1994). By contrast, we found that the use of the atmospheric response function model yields deviations up to 73% for the cumulative CO_2 uptake as compared to results obtained with the Princeton-GFDL OGCM (Joos et al., 1996).

2.3 Biospheric decay response function model

Nonlinearities in carbon cycle models arise also from the description of the potential carbon storage on land due to elevated CO_2 and nitrogen supply. Net primary production (npp) is usually described as a non-linear process. On the other hand, the decay of living and dead biomass back to atmospheric CO_2 is often described as a linear process. We can therefore represent biosphere models by applying equations describing the decay of organic matter and net primary production. The net carbon storage, δf_{ab}, which is the difference between additional photo-synthesis, δf_{npp}, and decay, δf_{decay} is then described:

$$\begin{aligned} \delta f_{ab}(t) =& \delta f_{npp}(t) - \delta f_{decay}(t) \\ =& \delta f_{npp}(t) \\ & - \int_{t_0}^{t} \delta f_{npp}(t') \cdot r_{decay}(t - t') \cdot dt' \end{aligned} \qquad (5)$$

The decay response function, r_{decay}, can be determined by allocating a unit of carbon into the assimilation pools of a biosphere model and then monitoring the back-flux into the atmosphere. It describes how long the added carbon remains in the biosphere.

In the case of CO_2 fertilization often simple logarithmic or Michaelis-Menton type relationship are used to describe the relationship between npp enhancement and atmospheric CO_2 (e.g., Friedlingstein et al., 1995; King et al., 1995). For example:

$$\delta f_{npp}(t) = npp(t_0) \cdot \beta \cdot ln\left[\frac{CO_{2,a}(t)}{278 ppm}\right] \qquad (6)$$

where $npp(t_0)$ is the pre-industrial net primary production, β a coupling factor (typically β=0.3) and 278 ppm the pre-industrial CO_2 concentration.

Figure 3 shows the biospheric decay response function as determined for a four box biosphere where each reservoir has a distinct overturning time (2.2, 2.9, 20, and 100 years) (Siegenthaler and Oeschger, 1987). The return flux of carbon into the atmosphere after a pulse injection into the assimilation pools increases after the
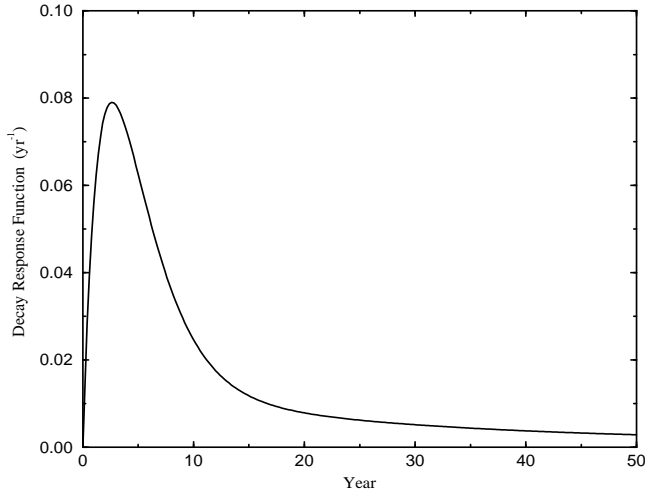
Figure 3. Decay response function for a 4-box biosphere (Siegenthaler and Oeschger, 1987). The response represents the return flux of carbon into the atmosphere after an initial pulse of carbon assimilation at time t=0. The maximum return flux is 2.7 years after the initial pulse input. It takes several hundred years, i.e. the overturning time scale of the slowly decaying soil pool, until all the carbon is returned to the atmosphere and equilibrium is reached.

time of injection (t=0) to reach its maximum about 2.7 years later. The return flux then disappears after several hundred years. The area under the curve corresponds to the fraction that leaves the biosphere during a certain time interval.

In summary, one can represent a complex, state of the art carbon cycle model to run scenario calculations linking atmospheric CO_2 concentration and anthropogenic emissions by using only 6 equations. The information of the complex model is transfered to the substitute model by the mixed-layer pulse response function and the biosphere decay response function. Non-linearities in the carbonate chemistry and in npp formulations are described by separate equations.

3   Global temperature change due to greenhouse gas forcing

The world ocean has a considerable heat capacity. This thermal inertia leads to significant deviations between the transient response of the climate system to a radiative perturbation as compared to an atmosphere in radiative equilibrium at each time. To assess the impact of greenhouse gases and other forcing agents the use of coupled atmosphere - ocean models is therefore necessary. A task which requires a substantial amount of computing power. For scenario calculations linking radiative forcing and the change of global surface temperature, the transient response of a coupled atmosphere-ocean model can be represented by prescribing its equilibrium sensitivity to a perturbation in the radiation budget and by calculating the oceanic heat uptake using mixed-layer

response functions described in Sec. 2.2. This approach is valid as long as temperature perturbations are not large enough to substantially change the ocean circulation and thus the surface-to-deep mixing as characterized by the mixed-layer pulse response function. In other word, the approach can be used as long as the climate system behaves in an approximately linear way. Substantial re-organization of the ocean circulation may occur when approaching an anthropogenic radiative forcing which corresponds to a doubling of pre-industrial CO_2 concentration (Manabe and Stouffer, 1993).

We have coupled the climate module described by Siegenthaler and Oeschger (1984) to the mixed layer pulse response function model. In this climate module the equilibrium temperature perturbation, δT_{eq}, is prescribed according to the results of atmosphere climate models. Typically, the equilibrium sensitivity is in the range of 1.5 to 4.5 K change in global surface temperature for a radiative forcing corresponding to a doubling of pre-industrial CO_2 concentration. Following Shine et al. (1990), the change in radiative forcing due to an increase in atmospheric CO_2 is described by a logarithmic relationship and a coupling factor of 6.3 W m^{-2} is used. Thus, radiative forcing as used here is the increase in the net radiation flux at the tropopause after allowing the stratosphere to come into a new equilibrium while keeping the surface-troposhere temperatures unperturbed. The perturbation in the sea surface temperature, δT_s, corresponds to the perturbation in surface air temperature. The input of heat into the surface layer, f_{as}, is described as:

$$f_{as} = \frac{1}{a_{oc}} \cdot 6.3 \frac{\text{W}}{\text{m}^2} \cdot ln \left[\frac{CO_{2,a}}{278ppm} \right] \cdot \left[1 - \frac{\delta T_s}{\delta T_{eq}} \right] \quad (7)$$

a_{oc} is the area fraction of the earth covered by the ocean (0.71). The change in the surface layer temperature is then again obtained as a convolution integral:

$$\delta T_s = \frac{c}{h} \int_{t_0}^{t} f_{as}(t') \cdot r_s(t - t') \cdot dt' \quad (8)$$

The constant c is introduced to convert units of Watt into temperature units by considering the heat capacity of sea water.

We have calculated the transient response of the pulse substitute model using the response function as obtained by the HILDA model (Siegenthaler and Joos, 1992) to a sudden doubling of the atmospheric CO_2 concentration. In Fig. 4 we compare the temperature response of the pulse model with results obtained with the Hamburg Large Scale global ocean circulation model for a very small step-function increment in the CO_2 concentration (but scaled to an equilibrium temperature change of 2.5 K) (Hasselmann et al., 1996). In the Hamburg ocean model surface-to-deep transport is slower and thus the
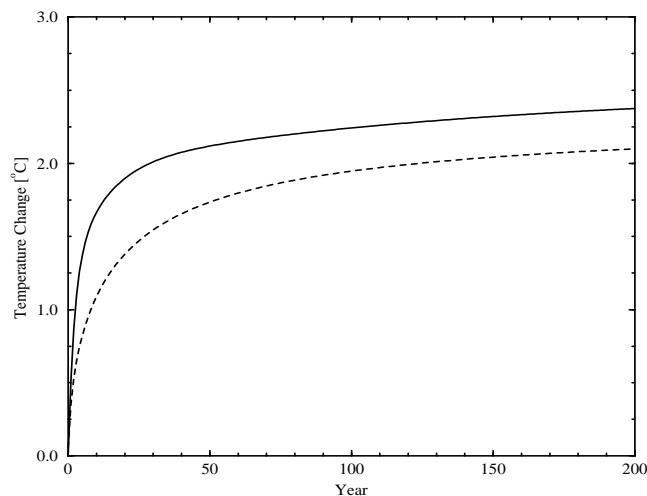
Figure 4. Transient temperature response to a doubling of the pre-industrial atmospheric CO_2 concentration at time t=0 as simulated by the pulse substitute climate model (dashed) using the HILDA ocean model and by the Hamburg Large Scale global ocean circulation model (solid) as given by (Hasselmann et al., 1996). In both models equilibrium temperature change for a CO_2 doubling is 2.5 K. The surface-to-deep mixing in HILDA is faster than in the Hamburg model which yields a slower surface temperature response.

surface layer is heated up faster. The apparently very fast response of the Hamburg model might be due to an approximation of the output by Hasselmann et al. (1996) applying three time constants only (2, 12, and 138 years). We have also calculate the response of the pulse substitute model to an increase in atmospheric CO_2 by 1% yr^{-1}. We applied two different temperature sensitivities of 2.1 and 4.6 K for doubling of CO_2 corresponding to the range obtained by 10 atmosphere-ocean general circulation climate models (AOGCMs). After 80 year, the global average surface temperature increase is 1.6 K and 2.4 K as modeled by the substitute model. This compares very well with the corresponding results of nine AOGCMs in the range of 1.5 to 2.7 K (see Fig. 6.4 in Houghton et al., 1996).

4 Conclusions

We have applied pulse response functions to build substitute models of complex climate and carbon cycle models. The substitute models can be used to perform scenario calculations linking global temperature change, atmospheric CO_2 and anthropogenic carbon emission. We have used mixed-layer and biospheric decay response function to avoid the problem arising from non-linearities in air-sea exchange and in net primary production. The time scales of the carbon turnover in the biosphere are one key aspect to determine the amount of additional carbon sequestered on land. We suggest that decay response functions should be used to characterize and

compare different biosphere models. Pulse substitute models are relatively easy to code. They are cost-efficient using only limited amount of CPU time. For example, a 1000 year run of the climate model described above needs 25 seconds on a workstation (DEC 2100/500). Various time scales in the range of 0 to 1000 years determine the response of atmospheric CO_2 and of the global temperature perturbation to the anthropogenic carbon emissions. This becomes evident when viewing the Figs. 1, 3, and 4. Attempts to describe this complex system by one single time constant (e.g., Nordhaus, 1992; O'Neill et al., 1994; Peck and Teisberg, 1992; Starr, 1993) can lead to misleading results and are unjustified.

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